Land Use, Biodiversity, and the Theoretical Structure of a Sustainability Indicator

by

Alfred ENDRES and Volker RADKE


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Abstract: We focus on the effects of deforestation for agricultural purposes on biodiversity. These have been dealt with in the recent literature treating 'forested land' and 'biodiversity' as synonyms. Opposed to that, this paper distinguishes between 'forested land' and 'forests' itself, the latter being interpreted as a measure of biodiversity. The regenerative capacity of forests is modeled as a function of the own stock and of the habitat size. In particular, the threat of a given minimum viable population to be achieved in the course of the reduction of habitats is taken into account. The corresponding structure of a sustainability indicator is elaborated.

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Address: University of Hagen, Institute for Economic Theory, P. O. Box 940, D-58084 Hagen, Germany. www.fernuni-hagen.de/VWLWTH/. Phone: ++49 2331 987 301. Fax: ++49 2331 987 302. E-mail: alfred.endres@fernuni-hagen.de
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1. Introduction

The problem of biodiversity decline - representing a reduction in natural wealth - has been identified to be one of the most severe impediments on arriving at sustainable development.\(^1\) Hartwick (1995) emphasized the role that the conversion of 'forested land' into 'land in agriculture' plays in the process of biodiversity decline.\(^2\) Within a model of intertemporal maximization of social welfare Hartwick computes a corrected version of net national product (NNP) accounting for the effect of agricultural land development on biodiversity. In the sense that NNP "is the amount that might be consumed which leaves 'capital' intact" (Hartwick 1994, p. 256) it could be interpreted as an indicator of sustainability.

However, Hartwick does not explicitly model the dependence of biodiversity from land utilization. Instead, 'forested land' and 'biodiversity' are taken to be synonyms for the sake of simplicity. Opposed to that, Swanson (1994a, 1994b) introduced the idea that land as a base resource affects the regenerative capacity of biological resources (e.g., forests) growing there. Swanson, however, is not interested in developing a sustainability indicator.\(^3\)

The present paper integrates the two afore mentioned approaches. Opposed to Hartwick (1995), we distinguish between 'forested land' and 'forests' itself. The idea that, in addition to the stock of forests, the size of habitats influences the regenerative capacity of forests is incorporated into the basic version of Hartwick's model (see Hartwick 1992; 1993a). In particular, the threat of a given minimum viable population to be achieved in the course of the reduction of habitats is taken into account.

\(^1\) For recent results see Perrings et al. (1995a; 1995b).

\(^2\) For related analyses see Hartwick (1992; 1993a; 1993b) and Hung (1993).

\(^3\) The same holds for Rowthorn/Brown (1995, p. 28) who distinguish between the variables 'undeveloped land' and 'number of species', the latter being a function of the former.
The structure of a sustainability indicator derived from the modified model is presented. Sustainable development is interpreted as a constant social welfare over time which presupposes the conservation of social wealth. Given the model characteristics sketched above, wealth conservation is achieved by combining the notions of 'strong sustainability' and of 'weak sustainability' to a criterion that we call *bounded weak sustainability* (BWS). This is done by using the concept of 'critical natural wealth' manifesting itself in minimum viable populations of certain species which are threatened by agricultural land development. Those critical stocks of biological resources define the 'ecological corridor' of the economic process. This corridor has to be embodied with a priority claim into a sustainability indicator. Within the bounds of this corridor, a weak sustainability rule is applied. The latter demands the conservation of aggregate wealth. We argue that the appropriately valued changes of all components of social wealth should sum up to zero at each time over the entire planning horizon. It is shown which kind of empirical data would be necessary to compute the resulting two-stage sustainability indicator. In particular, besides the physical data needed the problem of appropriate value components of the indicator is discussed. By this means, the paper develops a theoretical guideline for future empirical work on an indicator of sustainable development taking into consideration the linkage between land development and biodiversity.

The paper is organized as follows. In section 2, a modified version of Hartwick's basic model (see Hartwick 1992; 1993a) is presented. The theoretical structure of the model specific sustainability indicator is outlined in section 3. Empirical aspects are discussed in section 4. Section 5 concludes.

2. The Model

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4 This is closely related to the term 'critical natural capital' used by Pearce/Atkinson (1993; 1995). However, Pearce/Atkinson (1995, p. 170) "describe that natural capital which has such characteristics as to imply a high [total economic value] and hence nonsubstitutability as exhibiting *criticality*". Opposed to that, first we deduce, in section 3 below, the characteristic of 'nonsubstitutability' immediately from a formal criterion of sustainability and second we tie it to a certain stock level of the natural resource in question.
Consider the total land area, $A$, of the economy under consideration to be given and invariant in time. At each time $t$ there are two alternative uses for every single hectare. Either it can be devoted to 'agricultural' uses, or it is left in an 'natural' state, say forested. Denoting agricultural land by $L$, the residual $A - L$ is the base resource for forests. The latter host a variety of animal and plant species. Hence, the stock of forests, $B$, will be interpreted as a measure of biodiversity hereafter. In principle, it is possible to shift the borderline between these two categories of land use in both directions, where $X > 0$ is land development for agricultural purposes if $X > 0$ and reforestation if $X < 0$, respectively.

Following Swanson (1994a; 1994b) we assume that growth of forests, $G_t$, is a continuously differentiable and time invariant function $G: R^2_0 \rightarrow R$ of the stock of forests itself and of the habitat size as well:

\[(2-1) \quad G_t = G(B_t, A - L_t), \forall t.\]

The characteristics of the regeneration function underlying our subsequent analysis are depicted in figure 1 for different habitat sizes at time $t$. As a point of departure we consider the shape of the regeneration curve labelled '$G^1$'. This one is taken to hold for a given habitat size $A - L^1$. Here, $B^1_t$ is the corresponding saturation level while $B$ is the minimum viable population. Growth is zero for $B = 0$, $B = B$ and $B = B^1$, positive for $B < B < B^1$, and negative for $B > B^1$. Growth is also negative for $0 < B < B$. Hence, if the stock of forests once falls short of the level $B$ species relying on

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5 Here, we retain Hartwick's (1995) terminology. However, the term 'agricultural use' should be interpreted as a *pars pro toto*, e. g., for land use in industrial production or housing development as well.

6 At least in the tropics and also in Central Europe, forests are the most 'natural' occupation of soil.

7 Though it may be arguable whether the concept of a minimum viable population is 'realistic' in the case of forests, we refer to our assumption that forests are to be interpreted as a measure of *biodiversity*. For many species living within forests the assumption of a minimum viable population seems plausible. Hence, our assumption is to be interpreted in the sense that many species become extinct if the stock of forests falls short of a certain level.
forests will be extinguished and biodiversity will completely vanish in finite time (i.e., \( B \to 0 \) in finite time).

To capture the role of land as a base resource of forests we will assume that the saturation level is a function of the size of the habitat, \( A - L_t \). If there can exist, in the undisturbed ecological equilibrium, ten full-grown trees on one hectare, it will be 20 trees on two hectares. However, we assume the minimum viable population \( B \) to be exogenously given.\(^8\) The effect of a rise in \( L_t \) from \( L_{t1} \) to \( L_{t2} \) is shown in figure 1 as a decline in the saturation level from \( B_{t1} \) to \( B_{t2} \) and as a 'contraction' of the regeneration curve for all \( B > B \) (see the new shape labelled 'G\(^2\)').

Insert figure 1.

Note that there is an important implication of our assumption that the saturation level declines with a declining habitat size while the minimum viable population \( B \) is exogenously given (i.e., independent of the size of the habitat): It follows that there is a maximum size of land in agriculture, say, \( \bar{L} \) (resp. a minimum habitat size, \( A - \bar{L} \)) which equates saturation level and minimum viable population of the biological resource. Hence, if the habitat size falls short of \( A - \bar{L} \) the stock of the biological resource must eventually decline to zero.

We now turn to the description of the 'economic' side of our model. Agricultural production takes place according to

\[
(2.2) \quad Y_t = Y(K_t, H_t, L_t), \quad \forall t,
\]

where \( Y_t \) is output, \( Y: \mathbb{R}^3_{t0}^+ \to \mathbb{R}_0^+ \) is a time invariant and continuously differentiable production function, \( K_t \) is the capital stock and \( H_t \) is the aggregate input flow of timber harvested from the stock of forests, \( B_t \). We assume that timber is an essential factor of production in the sense that \( Y(K, 0, L) = 0 \).

For social welfare \( U_t \) we assume

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\(^8\) Swanson (1994a, p. 812, fig. 3) implicitly assumes that the minimum viable population is a (decreasing) function of the habitat size as well. For simplicity, we do not consider this case.
Here, \( U : \mathbb{R}^2_0^+ \rightarrow \mathbb{R}^+_0 \) is a time invariant and continuously differentiable social welfare function and \( C_t \) is aggregate consumption of private goods from agricultural production at time \( t \). We assume consumption of agricultural products to be essential in the sense that \( U(0, B) = 0 \).

The aggregate stock of forests, \( B_t \), is to be interpreted as a public good. Individual land owners derive utility from the total size of forests (not just from the size of the forest growing on their own forested land) because neighbours’ forests serve as a habitat for many animal and plant species as well as own forests. The implications of this assumption for government's behavior will be discussed in section 3 below.

Denoting by a dot above any variable its time derivative, capital investments are given by

\[
(2-4) \quad \dot{K}_t = Y_t - C_t - z(H_t) - w(X_t), \quad \forall t.
\]

In (2-4), the time invariant and continuously differentiable functions \( z : \mathbb{R}^+_0 \rightarrow \mathbb{R}^+_0 \) and \( w : \mathbb{R} \rightarrow \mathbb{R}^+_0 \) represent the costs of timber harvest and of land development, respectively. If \( X_t < 0 \), \( w(X_t) \) is the cost of reforestation.

There are two further state equations

\[
(2-5) \quad \dot{B}_t = G_t - H_t, \quad \forall t,
\]

\[
(2-6) \quad \dot{L}_t = X_t, \quad \forall t,
\]

where (2-5) describes the time path of the stock of forests depending on growth and harvest. From (2-6), the time path of agricultural land is determined by land development, where both deforestation and reforestation are possible.

The non-negativity constraints are:
However, because reforestation of land once agriculturally used is principally possible (that is, land development $X_t$ can be negative),

\[(2-8) \quad A - L_t \geq X_t \geq -L_t, \quad \forall t\]

holds. From (2-8), the area currently deforested at time $t$ cannot exceed the size of forested land, $A - L_t$, while the area reforested at time $t$ cannot exceed the size of agricultural land, $L_t$.

Starting our analysis at time $t = 0$ in sections 3 and 4 below, there are initial conditions

\[(2-9) \quad K_0 > 0, \quad B_0 > B > 0, \quad A > L_0 > 0.\]

We now turn to the description of the model specific sustainability indicator.

3. Designing a Sustainability Indicator

For a social planner adopting an infinite time horizon starting at $t = 0$, two responsibilities in the context of the model of section 2, above, are to be distinguished for analytical purposes.

First, for social optimality in a utilitarian sense the planner would have to internalize the externalities mentioned in section 2 above. In our model, the externalities requiring government intervention stem from the following divergency: Individual land owners will certainly recognize the effect of their own timber cut and land development on their own stock of forests, thus on total stock

\[9\] We use this term in the sense that the planner would accept the individual calculus of discounted utility maximization irrespective of the resulting time path of social welfare. While discounted utility maximization is consistent with eventually declining welfare (see Pearce/Atkinson 1995, p. 166), we will focus on sustainable development defined as constant welfare in section 3 below.
of forests, B, and, hence, on their own utility. However, they will neglect the fact that their individual stock of forests as a part of B generates utility for all other individuals as well (who, by assumption, cannot be excluded from consumption). Thus, individual land owners do not account for the fact that their own timber cut and land development creates a negative externality on all other individuals. Or, in other words, the existence of their own stock of forests generates a positive externality on all other individuals. Internalizing these externalities may succeed by a Pigovian taxation of land in agriculture (see Hartwick 1995).

Second, given successful internalization the planner may be interested in evaluating a competitive path according to the criterion of sustainable development. We define sustainable development as a constant and positive social welfare over the time interval \([0, \infty]\). Hence, an appropriate sustainability indicator is required.

There are two distinct fundaments for a sustainability indicator available from the literature. First, following Hartwick (1977) and the generalization of his analysis by Dixit/Hammond/Hoel (1980), constant welfare presupposes the conservation of wealth over the entire planning horizon. If wealth consists of man-made and natural assets, conservation of aggregate wealth is achieved by compensating for any decline in natural wealth by accumulation of man-made capital.\(^{10}\) This is the 'weak sustainability-rule' which has been used as the basis of a sustainability indicator (see Pearce/Atkinson 1993, 1995). Second, a 'strong sustainability-rule' is advocated in the literature. This rule demands the conservation of natural wealth irrespective of investments in man-made wealth (see, e. g., Pearce/Atkinson 1995, p. 170).

In our opinion, both, the weak sustainability-rule and the strong sustainability-rule are deducible from our definition of sustainable development only under unrealistically restrictive assumptions concerning the substitutability between man-made and natural wealth. Weak sustainability were a prerequisite for a constant and positive welfare if it were possible always to substitute for the reduction of a further unit of natural wealth by accumulation of man-made wealth irrespectively of the given stock levels (given the zero level ist not reached). Strong sustainability were a prerequisite for a constant and positive welfare if it were never possible to substitute for any

\(^{10}\) This holds as long as the zero level of natural wealth is not reached.
unit of natural wealth taken away by accumulation of man-made wealth, irrespective of the given stock levels.

Avoiding such restrictive assumptions, we will deduce from the goal of a constant and positive welfare a criterion which we will call bounded weak sustainability (BWS). This criterion emphasizes limits of substitutability which arise at certain stock levels of single identifiable components of natural wealth. Above those levels, however, substitutability prevails. We will operationalize this criterion by introducing the notion of 'critical natural wealth'.\(^{11}\) We define as critical a positive level of a stock of a natural resource if reaching this level inevitably will lead to an unsustainable development.

Note that our BWS criterion comprises the notions of weak sustainability and of strong sustainability as special cases. Interpreting, at time t, for each component of natural wealth the actual stock level as critical would end in strong sustainability.\(^{12}\) Denying the existence of a critical stock level for any component of natural wealth would end in weak sustainability.

In the model of section 2 above, the minimum viable population of the biological resource, \(B\), represents 'critical natural wealth'. Would the stock once fall short of this level, it must eventually decline to zero. In this case, agricultural production would no longer be possible because by assumption harvest of the biological resource is an essential factor of production. Consumption must decline to zero (possibly, after the remaining capital stock \(K\) had been consumed). Consequently, welfare must decline to zero as well because, by assumption, \(C_t\) is essential. This, however, would violate our definition of sustainable development. Hence, the conservation of \(B\) is a necessary condition for sustainable development in our model.

However, this is not a crucial point under ideal circumstances. If the planner manages his internalization task perfectly, there is no reason to assume that \(B\) will be endangered along a competitive path at all. Conservation of \(B\) is, however, a task in its own right if perfect internalization is missed due to information deficits of the planner. In this case, the notion of a 'socially optimal path' is just a theoretical benchmark which is, however, unattainable under real

\(^{11}\) See footnote 4 above.

\(^{12}\) This would even be an intensification of the strong sustainability rule if the latter is interpreted to allow for substitution between heterogeneous components of natural wealth (see Pearce/Atkinson 1995, p. 170).
circumstances. Even if the planner designs a tax on land in agriculture according to his best knowledge there will be no guarantee that \( B \) will not be fallen short of. In this case, the notion of 'weak sustainability' à la Hartwick (1977) and Dixit/Hammond/Hoel (1980) is to supplemented by the conception of 'strong sustainability'. The latter is introduced by using the concept of 'critical natural wealth' constituting our BWS criterion.

In our view, critical stocks of natural resources define the 'ecological corridor' of the economic process. If neither the price mechanism nor (imperfect) internalization taxes can guarantee for the preservation of critical stocks, this corridor has to be embodied with a priority claim into a sustainability indicator. Hence, our indicator comprises, in first priority, an index number \( I^1 \):

\[
(3-1) \quad I^1_t := B_t - \underline{B} > 0, \; \forall t \geq 0.
\]

The index number \( I^1 \) reports the deviation of the actual stock of the biological resource from it's 'critical' level. For sustainable development, it has to be positive for all \( t \geq 0 \).

Within the bounds of the 'ecological corridor' defined by (3-1), a weak sustainability rule is applied. The latter demands the conservation of aggregate wealth. Wealth consists, in the present model, of the aggregate capital stock, \( K \), the aggregate stock of forests, \( B \), the latter representing a measure of biodiversity, and of the aggregate stock of land in agriculture, \( L \). Hence, the appropriate valued changes in the stocks of these three assets have to sum up to zero at each instant of time. Consequently, our indicator comprises, in second priority, an index number \( I^2 \):

\[
(3-2) \quad I^2_t := K^*_t + Y^*_{2t} B^*_t + w^*_t L^*_t = 0, \; \forall t \geq 0.
\]

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\(^{13}\) Note that the conservation of the minimum viable population of the biological resource requires the conservation of a certain minimum size of the habitat as well (see section 2 above). However, land development for agricultural purposes is not the only reason that might endanger the minimum viable population \( B \). Extensive timber harvest might be another reason. Hence, conservation of the minimum size of the habitat is a necessary but not a sufficient condition for the conservation of \( B \).

\(^{14}\) A proof that (3-2) guarantees \( \dot{U}_t^* = 0 \), for all \( t \geq 0 \), is given in the appendix.
Equation (3-2) represents a modified Hartwick rule (see Hartwick 1977; Dixit/Hammond/Hoel 1980). Here, $Y_{2t}$ is the partial derivative of the production function $U(2,2)$ with respect to its second argument (the same convention holds also for the functions $U$ and $G$ hereafter). $Y_{2t}^n := Y_{2t} - z_t'$ is the net marginal productivity of timber at time $t$ (i.e., gross marginal productivity minus marginal cost of harvest).

The superscript * indicates a competitive path given successful internalization (i.e., a socially optimal path in an utilitarian sense). As mentioned above, this is just a 'Nirvana' given uncomplete information of the planner. Hence, under real circumstances the sum in (3-2) will not 'correctly' report the change in social wealth. It will provide just an indication about the direction of change.

In the light of these caveats, it will be shown in section 4 below which kind of empirical data would be necessary to compute the two-stage sustainability indicator $(I_1^t, I_2^t)$. In particular, in addition to the physical data needed the problem of appropriate value components of the indicator is discussed.

4. Empirical Aspects

From (3-1) and (3-2) it follows that there are two distinct types of data required to compute our two-stage sustainability indicator $(I_1^t, I_2^t)$. These are

1. data concerning physical stocks and variations in stocks of all relevant wealth components and
2. data concerning the valuation of stocks and changes in stocks from 1.

Note that this holds for both, the first priority and the second priority of our sustainability indicator. In first priority, both, the actual stock, $B_t$, and the minimum viable population of the biological resource, $B$, would have to be quantified. This would allow for the determination of $I_1^t$. However, this is not a mere descriptive task which could be solved on the basis of physical data alone. Note that the minimum viable population $B$ is 'critical' only because the normative criterion of sustainability is necessarily violated if the stock $B$ would fall short of this level. This is so, in the context of the model presented in section 2, because the resource harvest is an essential factor of...
production and consumption is an essential argument of the welfare function. Hence, not the extinction of the biological resource \textit{per se} has to be valued negatively. However, the characteristics of the functions assumed in (2-1) to (2-3) are generating valuations which justify the statement that the extinction of the biological resource is intolerable given the normative criterion of sustainability. Thus, an empirical assessment of $I_t^1$ does not require physical data alone. Rather, a consensus on whether the minimum viable population $B_t$ is to be viewed as 'critical' or not has to be reached first.

Beyond the level $B_t$, in second priority, the physical changes in all the relevant components of social wealth have to be determined (i.e., $K_t^*$, $B_t^*$ and $L_t^*$). This is illustrated in equation (3-2). Moreover, appropriate valuation of these physical changes has to be carried out. To clarify matters, we convert equation (3-2) into\textsuperscript{15}

\begin{equation}
I_t^{2*} := (Y_{1t}^*)K_t^* + (Y_{21t}^* + Y_{22t}^*G_{11}^* + U_{21t}/U_{11t})B_t^* + (Y_{32t}^* + \omega_t^* - Y_{22t}^*G_{22}^*)L_t^* = 0, \quad \forall t \geq 0.
\end{equation}

The point illustrated by equation (4-1) is that, in second priority, our sustainability indicator does not require physical data only. Rather, the physical changes of heterogeneous components of social wealth have to be weighed against each other to determine the change of total wealth. The value components necessary for computing the index number $I_t^{2*}$ in our model are given in parentheses in (4-1). There are three important points.

1) The value of the \textit{non-marketable asset} 'biodiversity' (i.e., the marginal rate of substitution $U_{21t}/U_{11t}$) could be investigated by, e.g., contingent valuation methods. First, this would enable the planner to better fulfill his task of internalization of externalities as described above. Second, this would improve the informational content of the index number $I_t^{2*}$ concerning the criterion of sustainability because a better done 'internalization job' would 'shift' the economy closer to the social optimal path.

\textsuperscript{15} Solve, from the appendix, equations (A-14) and (A-16) for $Y_{21t}^*$ and $\omega_t^*$, respectively, and insert into (3-2) to obtain (4-1) after some manipulations. This is only a necessary condition for a constant welfare but nevertheless may serve to illustrate empirical necessities.
2) Market prices can serve as proxies for several value components. The marginal productivity of capital, $Y_{1t}^*$, could be approximated by the price of capital goods prevailing in the economy. Market prices could also serve to approximate the net marginal productivity of marketable timber, $Y_{2t}^{n*}$, and the corresponding price change, $\Delta Y_{2t}^{n*}$. The same holds for the marginal productivity of land in agriculture, $Y_{3t}^*$, and the change in marginal costs of deforestation (resp. reforestation), $\psi_t^*$. Note, however, that observable market prices also would be 'correct' only along a competitive path with successful internalization (i.e., along a social optimal path). Moreover, it has to be taken into account that under real circumstances the informational content of market prices may be biased due to a variety of reasons besides the existence of public goods, i.e., market power, governmental regulations, uncomplete information of agents.

3) Besides the 'economic' value components mentioned above, knowledge from the natural sciences is a constituent part of the value components in equation (4-1). First, the marginal contribution of the own stock of the biological resource to its regeneration, $G_{1t}^*$, has to be known. Concerning this, there has already been considerable effort in the literature on resource economics (see Clark, 1990, and the references cited there). Second, the marginal contribution of the habitat size to the regeneration of the biological resource, $G_{2t}^*$, has to be investigated. Up to now, this second point seems quite less familiar to resource economists. Note, again, that these biological value components are 'correct' only along a social optimal path.

5. Conclusions

The present paper intended to contribute to the theoretical foundation of future empirical work on an indicator of sustainable development taking into consideration the linkage between land

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16 The "Theory of island biogeography" of MacArthur/Wilson (1967) might serve as a point of departure of future research (see Rowthorn/Brown 1995, p. 28). MacArthur/Wilson, however, observed a log-linear relationship between the area of islands and the number of species.
development and biodiversity. There were two prominent aspects which are, already in the present state of work, of immediate policy relevance.

First, the idea of 'critical natural wealth' was integrated into the theoretical structure of a sustainability indicator. This idea was operationalized by the notion of a minimum viable population of a biological resource representing a measure of biodiversity. By this means, it was pointed out that there is an 'ecological corridor' indicating the boundaries within which the economic process must be confined.

Second, our paper accounts for the idea of an ecological corridor but nevertheless goes far beyond this. The mere respectation of the ecological corridor is necessary but not sufficient for sustainable development. It is not trivial what happens within the ecological corridor if sustainability is at hand. Rather, we have derived an additional rule which guarantees the conservation of social wealth as an aggregate of heterogeneous wealth components. Taking into consideration both, the boundaries indicated by the ecological corridor and the rule for wealth conservation simultaneously is sufficient for sustainable development. We call this 'bounded weak sustainability' (BWS).

In the context of our model it is important that within the ecological corridor any loss in 'biodiversity' (any loss in 'consumption') has to be compensated for by an equivalent increase in 'consumption' (by an equivalent increase in 'biodiversity'). Emphasizing this second aspect, in addition to the idea of an ecological corridor, our paper intends to initiate a quantitative discussion of the value components of a sustainability indicator in the context of the biodiversity problem. This may serve to rationalize the dispute on the politically relevant but not yet operationalized question of whether mankind 'consumes its wealth'. Should it turn out, in the light of the biodiversity problem, that the answer is 'yes' the discussion of value components may yield a system of information which allows for corrections of undesirable developments even within the ecological corridor.

Appendix
It will be shown that, within the bounds of the ecological corridor, (3-2) is sufficient for a constant welfare over the entire planning horizon. Below, we will not explicitly note the time variable where no confusion may arise. Differentiate (2-3) and (2-4) with respect to time to obtain

\[ \dot{U} = U_1 \dot{C} + U_2 \dot{B}, \forall t, \]  

\[ \dot{C} = \dot{Y} - \ddot{K} - z^\prime \dot{H} - w^\prime X, \forall t, \]

where \( \ddot{K} := d^2K / dt^2 \). Furthermore, from (2-2) it follows with (2-6) and with the modified Hartwick rule (3-2):

\[ \dot{Y}^\ast = -Y_1^\ast Y_2^n \dot{B}^\ast - Y_1^\prime w^\prime X^\ast + Y_2^\prime \dot{H}^\ast + Y_3^\prime X^\ast, \forall t \geq 0. \]

Differentiation of the modified Hartwick rule (3-2) with respect to time yields with (2-1), (2-5) and (2-6):

\[ \ddot{K}^\ast = -Y_2^\prime G_1^\prime \dot{B}^\ast + Y_2^\prime G_2^\prime X^\ast + Y_2^\prime \dot{H}^\ast - w^\prime X^\ast - w^\prime X^\ast, \forall t \geq 0. \]

Defining, for any variable \( x \), its growth rate as \( \dot{x} := x / x \), inserting (A-2), (A-3) and (A-4) in (A-1) yields after some manipulations:

\[ \ddot{U}^\ast = U_1 \dot{H}^\ast \left( Y_2^\prime - z^\prime - Y_2^n \right) 
- U_1 Y_2^n \dot{B}^\ast \left( Y_1^\prime - G_1^\prime - U_2^\prime / U_1 Y_2^n - \dot{Y}_2 \right) 
- U_1 w^\prime X^\ast \left( Y_1^\prime - Y_3^\prime / w^\prime + Y_2^n G_2^\prime / w^\prime - \dot{w}^\prime \right), \forall t \geq 0. \]

To complete the proof, we have to characterize a competitive path given successful internalization which is identical with a social optimal path. The latter is computed by a social planner solving...
\[
\max_{t \geq 0} \int_{1=0}^{\infty} e^{-\rho t} U(t) dt, 0 < \rho < \infty
\]

s.t. (2-4) to (2-9), where \(\rho\) is the social rate of discount. Here, we assume that the planner adopts the individual rate of time preference. Focusing on interior solutions (thus, not explicitly treating (2-7) and (2-8)) the current value Hamiltonian can be written as

\[
H = U(C, B) + \lambda (Y(K, H, L) - C - z(H) - w(X)) + \Omega(G(B, A - L) - H) + \Phi X.
\]

Here, \(\Lambda_t, \Omega_t\) and \(\Phi_t\) are the costate variables corresponding to the stocks \(K_t, B_t\) and \(L_t\), respectively. The necessary conditions for an interior solution (existence provided) are, besides (2-4) to (2-6), where the superscript * indicates a competitive path given successful internalization (i.e., a social optimal path):

\[
(A-6) \quad 0 = \frac{\partial H}{\partial C} = U_t^* - \Lambda^*, \ \forall \ t \geq 0;
\]

\[
(A-7) \quad 0 = \frac{\partial H}{\partial H} = \Lambda^* (Y_t^* - z^*) - \Omega^*, \ \forall \ t \geq 0;
\]

\[
(A-8) \quad 0 = \frac{\partial H}{\partial X} = -\Lambda^* w^* + \Phi^*, \ \forall \ t \geq 0;
\]

\[
(A-9) \quad \dot{\Lambda}^* = -\frac{\partial H}{\partial K} + \rho \Lambda^* = (\rho - Y_t^*) \Lambda^*, \ \forall \ t \geq 0;
\]

\[
(A-10) \quad \dot{\Omega}^* = -\frac{\partial H}{\partial B} + \rho \Omega^* = (\rho - G_t^*) \Omega^* - U_t^*, \ \forall \ t \geq 0;
\]

\[
(A-11) \quad \dot{\Phi}^* = -\frac{\partial H}{\partial A} + \rho \Phi^* = \rho \Phi^* - \Lambda^* Y_t^* + \Omega^* G_t^*, \ \forall \ t \geq 0.
\]

From (A-9) with (A-6) follows the Ramsey rule:

\[
(A-12) \quad \rho = \dot{U}^*_t + Y_t^*, \ \forall \ t \geq 0.
\]
From (A-7) with (A-6) and the definition of net marginal productivity of timber, \( Y_2^a := Y_2 - z^* \), it follows: \( \Omega^* = U_1^* Y_2^a \), for all \( t \geq 0 \). Inserted in (A-10) this yields:

\[
(A-13) \quad \rho = \hat{U}_1^* + \hat{Y}_2^a + G_t^* + \frac{U_2^*}{U_1^* Y_2^a}, \quad \forall t \geq 0.
\]

Equate (A-12) and (A-13) to obtain:

\[
(A-14) \quad \hat{Y}_2^a + G_t^* + \frac{U_2^*}{U_1^* Y_2^a} = Y_1^*, \quad \forall t \geq 0.
\]

For land in agriculture it follows from (A-8) with (A-6): \( \Phi^* = U_1^* w^* \), for all \( t \geq 0 \). Inserted in (A-11) one obtains with (A-6), (A-7) and the definition of the net marginal product of timber:

\[
(A-15) \quad \rho = \hat{U}_1^* + \hat{w}^* + \frac{Y_1^*}{w^*} - \frac{Y_2^a G_t^*}{w^*}, \quad \forall t \geq 0.
\]

Equating (A-12) and (A-15) yields:

\[
(A-16) \quad \frac{Y_1^*}{w^*} + \hat{w}^* - \frac{Y_2^a G_t^*}{w^*} = Y_1^*, \quad \forall t \geq 0.
\]

Equation (A-14) represents a condition for an intertemporal efficient utilization of the biological resource. Equation (A-16) is the analogon for land in agriculture. Both conditions serve to equalize the return of investments in the stock of the biological resource and in agricultural land, respectively, with the return on capital investments.

With (A-14) and (A-16), on the RHS of (A-5) the second and the third terms in parentheses sum up to zero. Moreover, the first term in parentheses sums up to zero by definition. Hence, (3-2) is sufficient for \( \hat{U}_1^* = 0 \), for all \( t \geq 0 \).

References


Figure 1

\[ L_1^1 < L_1^2 \]

\[ G^1 \]

\[ G^2 \]

0

B

\(\overline{B}\)

\(\overline{B}^2\)

\(\overline{B}^3\)