The Role of Spatial Externalities in the Evolution of Urban Sprawl

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Abstract: A dynamic model of a land developer’s conversion decision is constructed, in which spatial externalities influence the expected value of a parcel in alternative uses and lead to interaction effects over time among developers. Using theory and simulation results, we show that for a critical range of positive and negative externalities, these effects lead to the evolution of a scattered, noncontiguous pattern of residential subdivision development that is qualitatively similar to the observed pattern of residential subdivisions development in many U.S. exurban areas. The hypothesized presence of these interaction effects has implications for the dynamic evolution of land use pattern and growth management policy.
1. Introduction

Land use and environmental quality are intricately linked. For example, land use pattern is a large determinant of nonpoint source pollution of ground and surface water. Contaminants generated from human activities on the land drain into aquatic systems and negatively impact water quality and aquatic habitats. Agriculture has been identified as a major nonpoint source of nutrients, but the increasingly visible alternative to agriculture—development of one form or another—may pose equal, if not more serious threats to these ecosystems. Runoff during construction, increases in impermeable surfaces, suburban use of fertilizers and pesticides, and failed septic systems are factors associated with many types of urban development that can contribute to increased levels of pollutants in aquatic ecosystems.

While the amount and type of human activity in a watershed is important, so is its spatial distribution. Development patterns in the United States have changed significantly over the past few decades. In particular, the migration of people, industries, and businesses to suburban and exurban areas has resulted in the emergence of low density, noncontiguous settlement patterns, commonly referred to as “sprawl” or “leapfrog” development. These changing land use patterns have received significant attention from the public and policy makers concerned with both the environmental consequences of such development and the increasing public service costs associated with a more decentralized population.

The design of effective growth management policies requires an understanding of the economic, institutional, and social forces that underlie individuals’ location decisions. Towards this end, we seek an understanding of the spatial and temporal evolution of noncontiguous, sprawl patterns of residential development. Specifically, the focus here is on residential land use conversion in U.S. exurban areas at the suburban-rural fringe. These are areas that currently have the highest rates of loss of agricultural and forested land, in addition to land in other natural states (e.g. wetlands). These also tend to be areas in which local governmental jurisdictions have less stringent land use controls, e.g. many times large tracts of forested or agricultural lands are zoned as residential use with a certain (typically low) maximum development density. Therefore we are interested in understanding the evolution of sprawl development patterns that are typical of many U.S. exurban areas, given that they are not fully determined by zoning and other land use constraints. It is hypothesized that, in addition to a household’s location relative to fixed, exogenous features of the landscape (e.g. location of employment centers, waterfront, roads), households’ locations relative to each other also matter. This interdependence is hypothesized to arise from spatial externalities that are generated by the land uses of neighboring land parcels and which influence the relative value of a land parcel in alternative uses. Therefore spatial externalities are seen as creating an interaction among individuals’ location decisions over time, such that an individual’s residential location decision in one period may influence the decision of another in the next period.

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3 “Exurban” refers to areas that are relatively far from urban centers and largely rural in nature, but also contain some mix of urban uses.
The paper is organized as follows. After a brief discussion of land use conversion in exurban areas, we motivate the modeling approach taken here by contrasting recent models of interacting agents, including models of industrial location developed by Arthur (1988) and Krugman (1996), with more standard models of residential location found in the urban and regional economics literature. We argue that a Krugman-like approach, in which individual agents making locational decisions interact via spatial spillover effects, captures our notion of how spatial externalities may influence the evolution of a sprawl pattern of development. A dynamic model of a land developer’s conversion decision is constructed, in which spatial externalities are incorporated into the developer’s decision.\(^4\) In doing so, we borrow from techniques that originate from statistical mechanics models of interacting particle systems and which have recently been applied in the economics literature to model social interactions (see Durlauf, 1996, for a review of recent models). Using theory and simulation results, we show that for a critical range of positive and negative spatial externalities, these effects lead to the evolution of a scattered, noncontiguous pattern of residential subdivision development that is qualitatively similar to the pattern of observed pattern of residential subdivisions development in our Maryland study area. The paper concludes with a discussion of the implications of spatial externalities for the evolution of land use pattern and growth management policy.

2. Land use conversion in exurban areas

The amount of development in traditionally rural areas in the U.S. has increased substantially in the recent past. Many of the fastest growing counties in America in the past 5-10 years are those located in exurban areas -- areas that are relatively far from urban centers and largely rural in nature, but also contain some mix of urban uses. Overwhelmingly, the largest increase in land use in these areas has been in residential development. Because of the almost exclusive conversion of land to residential use in these areas, this research is primarily concerned with the conversion of undeveloped land in exurban areas, e.g. agricultural and forest lands, to residential use.

For example, consider our study area in central Maryland, which is comprised of the counties encompassing the watershed of the Patuxent River, a major tributary of the Chesapeake Bay (see Map 1). The observed spatial pattern of residential parcels in the exurban portions of this area can be described in terms of both a clustering and scattering of residential development. Alternatively, we say that this pattern exhibits both positive and negative spatial correlation of residential development. For example, Map 2 shows a closer look at the spatial pattern of residential development in one of these exurban counties, Calvert County, which show the scattered clustering of recent residential subdivision development. The clustering of multiple residential lots within one residential development or subdivision can be understood in terms of the developer’s profit-maximizing behavior. Because the profits from the subdivision and sale of individual residential lots are increasing in the density of development, developers will generally choose to subdivide the property into as many lots as is permitted by zoning and other land use regulations. It is trivial to note that residential lots originating from the same

\(^4\) The model is amenable to estimation and I hope to have some preliminary estimation results as part of the paper presentation at the World Congress.
undeveloped piece of land are positively correlated in space. On the other hand, these residential subdivisions are usually not contiguous, but rather are scattered throughout the exurban areas. In many cases, the pattern of subdivision development follows the location of exogenous landscape features, e.g. roads, public services, and waterfront. However, conditional on zoning constraints and exogenous sources of heterogeneity, residential subdivisions are many times negatively correlated with each other, as evidenced by Map 2.

3. Models of Residential Location

The formation of urban land use patterns can be understood as a dynamic system comprised of many individual land developers distributed in space, each of whom owns a land parcel and makes profit-maximizing choices regarding the use of the parcel. The collective result of these actions is the dynamic evolution of a regional land use pattern over time and space. Agents’ land use choices may not be independent of each other due to the existence of spatial externalities generated from neighboring agents’ land use decisions. Therefore we are interested in characterizing the emergence of a large-scale regional land use pattern as a result of the small-scale interactions among many individual land use agents. In addition, we seek an approach in which key sources of heterogeneity can be taken into consideration, namely the heterogeneity of the landscape which creates heterogeneity among the bundle of attributes associated with different land parcels.

This approach to modeling land use conversion differs significantly from most of the standard residential location models found in the economics literature. For example, monocentric models (Alonso, 1964; Muth, 1969; Mills, 1972) describe a long run spatial equilibrium pattern of land uses given the existence of an exogenously determined central business district to which residents commute. Because the location of the city center is not endogenous to residential location, these models are essentially a static representation of an equilibrium land use pattern for either a fixed population or utility level. Many extensions to the basic monocentric model have been developed, including policentric models and dynamic models that seek to explicitly model the process of urban growth by considering the durability of housing and intertemporally optimal decision making. In particular, Mills (1981) develops a dynamic monocentric model, in which temporary “leapfrog”

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5 The relationship between public infrastructure, e.g. roads and public sewer and water lines, and residential land use patterns is an important subject in its own right. It is clear that over time, the provision of these services and infrastructure is endogenous to residential location decisions. However, because the focus of this paper is on the endogenous interactions among agents making residential location decisions, we abstract from the endogeneity of these features of the landscape and treat them as exogenous. Over a short time horizon, this is not an unreasonable assumption.

6 An exception are the dynamic models of residential location in the regional economics literature (for a review, see Haag, 1989). Related to this approach is Werczberger’s (1988) regional model of urban land use change with endogenous spatial externalities. His notion of the dynamic role of spatial externalities in influencing individuals’ location decisions is very similar to ours, but his model is developed at an aggregate regional level.

7 These models allow for more than one urban center (see Richardson, 1988, for a discussion of these models). While these models are clearly a more realistic representation of today’s metropolitan areas, they share some of the same limitations as the monocentric model. For example, most retain assumptions about exogenously located centers. In addition, accessibility to urban centers remains the determining factor in land rents.
development results due to the speculative behavior of heterogeneous developers. However, over time this type of model predicts an infill of development rather than an increase in the fragmentation of the development pattern. This is contrary to what we witness in our study area, in which new subdivision development increases rather than decreases the fragmentation of the landscape (see Map 2).

Models of Interacting Agents

In developing a spatially disaggregate model of land use conversion with interacting land developers, we draw on models of interacting economic agents, many of which have been adapted from statistical mechanics models of interacting particle systems. These models provide an approach to linking processes that interact on a micro-level with the collective properties of the system. For this reason, they have proven useful in modeling the evolution of economic systems comprised of many, heterogeneous and interdependent agents. Applications of this approach in economics include models of industry location (Krugman, 1996; Arthur, 1988a, 1988b), employment status (Topa, 1996), business cycles (Durlauf, 1994), asset price formation (Brock and Hommes, 1995), and social pathologies, e.g. crime (Glaeser, Sacerdote, and Scheinkman, 1996). In addition, agent-based simulation models have been developed (see Epstein and Axtell, 1996, for a review) to study the evolution of a variety of human social phenomena, including trade, migration, group formation, interaction with the environment, cultural transmission, and disease propagation. Although these models are widely divergent in subject matter, their common feature is the modeling of interaction among many individual units at a highly disaggregate level, which leads to the emergence of collective behaviors and patterns in social and economic systems. This approach is appealing for modeling the evolution of land use pattern for several reasons. First, these models are spatially disaggregate and therefore each individual agent is identified with a set of neighbor agents. Second, the process is viewed as stochastic and interaction among individual agents is a function of the discrete choices made by agents. This characterization of the problem makes clear the similarity to standard economic models of discrete choice. The central difference here is the inclusion of a “social interaction” effect in the individual’s utility function. Third, there is some flexibility in terms of how the interaction effect may be specified. For example, Brock and Durlauf (1995) specify a global interaction, in which an individual’s decision is influenced by their expectation over average group behavior. Topa (1996) specifies a local interaction effect, in which the individual is positively influenced by the proportion of neighbors that are in an employed state. However, both approaches are concerned with a positive interaction effect among individuals, whereas the mix of positive and negative externalities among land uses may result in a more complex set of interactions among agents.

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8 An early example is Schelling’s classic study of racial segregation (1971), which is one of the first examples in economics of modeling the collective consequences of interactions among heterogeneous agents. Using a simple, spatially explicit model of agents’ locations in space, Schelling shows how small differences in agents’ preferences over the racial composition of their immediate neighborhood can lead to fully segregated patterns of residential location.
4. A Dynamic Model of Residential Land Use Pattern

In developing a model of land use conversion, we start from the viewpoint of a profit-maximizing agent who owns an undeveloped land parcel and makes a discrete choice in every period regarding the subdivision of a parcel for residential use. Conditional on the parcel being undeveloped in the present period, the agent’s decision is simplified to a binary choice of converting her parcel to residential use or keeping her parcel in an undeveloped use, such that the present discounted sum of all future expected returns from the land are maximized. If the developer chooses not to convert her land in the present period, it is with the understanding that an optimal conversion decision will be made in the future. Once converted, developers supply residential lots to households, who make location decisions by choosing a bundle of attributes associated with a particular location to maximize utility. The developer’s basic optimization problem is specified as:

\[
\Psi(i, t) = \max_{s(i,t) \in \{-1,1\}} \left\{ \Psi_{s(i,t)}(i, t) \right\}
\]

where \(\Psi_{s(i,t)}(i, t)\) is the developer’s expected present discounted value of parcel \(i\) in land use \(s(i,t)\), conditioned on \(s(i,t-1) = -1; s(i,t) \in (-1,1)\), where -1 represents an undeveloped state and 1 represents a residential state.

In order to capture the evolution of a regional land use pattern as the consequence of many developers’ profit-maximizing behavior, we consider a region comprised of many individual parcels of land, each of which is owned by an individual developer agent. Therefore the spatial distribution of agents across the region corresponds to the spatial distribution of land parcels across the landscape. Land parcels (and therefore agents) are arranged on a regular two-dimensional grid and are indexed by \(i = 1,\ldots,N\), which defines a unique location of the parcel in the region. Distance between any two parcels is measured as the distance between the nearest edges of parcels. Parcels that share a common border are said to be contiguous. For each parcel \(i\), various sets of neighbors can be defined according to the relative distance between parcels. In particular, the set of parcels that are contiguous to parcel \(i\) are defined as parcel \(i\)’s nearest neighbors.

Land is treated as a bundle of heterogeneous goods and therefore developers are assumed to formulate an expected value of a parcel in residential use by forming a hedonic on the current characteristics associated with their parcel. In addition, the developer is likely to form expectations over future changes in residential land prices by taking regional

Footnotes:
9 Here, undeveloped uses include agricultural and other resource production-oriented uses of the land, e.g. commercial forestry, as well as land in natural states.
10 In the reminder of the paper, we treat the undeveloped parcel as the unit of observation and therefore, the decision that is modeled is the developer’s decision to subdivide her parcel into multiple residential lots or to keep her parcel in an undeveloped use.
11 Because the focus of this research is on the spatial dynamics of interdependence among agents, we abstract here from the question of the developer’s optimal timing of conversion and assume simply that, if the developer chooses not to convert in the present period, then she will choose to develop at some optimal time in the future.
12 Agents are assumed to be homogeneous in all respects and consequently are differentiated only through the heterogeneity of their land parcels.
growth pressures into consideration (e.g. future population and income changes) and choosing an optimal time to convert based on her expectations. Here we abstract from the temporal aspects of individual developers’ optimal timing decisions and simplify the regional growth effects by assuming that the regional demand for new housing is constant. Therefore, given the bundle of characteristics associated with parcel i in period t, the developer calculates her expected value of parcel i and makes a decision to convert the parcel or leave it undeveloped at the beginning of the current period t. This implies a lagged effect, in which the spatial distribution of characteristics across the landscape in period t-1 determines the state of parcel i in period t.

In specifying the developer’s expected value function, a key distinction is made between the influence of landscape features that are considered to be exogenous characteristics of the landscape and the spatial externalities that are generated by the surrounding pattern of land uses within a defined neighborhood of parcel i. Because these externalities are generated by the neighboring land use pattern, these effects are clearly endogenous to the land use conversion process. Given this distinction, the developer’s expected value of parcel i in land use s in period t, \( V_s(i,t) \), is specified as a function of the following components: (1) the relative contribution of exogenous parcel attributes in period t-1, \( H_s(i,t-1) \), which includes the relative value of features specific to parcel i, e.g. size, vegetation, soil type, slope, services available to the parcel, and exogenously determined location features, e.g. distances to cities, shopping centers, and waterfront; (2) the net influence of spatial externalities generated from the land use in period t-1 of parcels located within a defined neighborhood of parcel i, which may either increase or decrease the value of parcel i in land use s in period t, \( I_s(i,t-1) \); and (3) a random component, \( \epsilon_s(i,t) \), which may include unobservable parcel characteristics present in period t-1 that influence the value of parcel i in period t. Taken together, the developer’s expected value of parcel i in land use s, given the parcel is not yet in a developed state, is:

\[
V_s(i,t) = (H_s(i,t-1), I_s(i,t-1), s(i,t) \mid s(i,t-1) = -1)
\]

where \( s \in \{-1,1\} \); \( H_s(i,t-1) = \alpha_s h(i,t-1) \), where \( h(i,t-1) \) is a 1xk vector of exogenous parcel characteristics, and \( s \) is a 1xk vector of parameter values, which determine the relative contribution of each characteristic to the value of parcel i in land use s.

Given the stochastic nature of the agent’s decision making process, the land use conversion process is described in terms of the transition probabilities of land parcels. The probability that agent i will convert her parcel from an undeveloped to developed state is equal to the probability that the expected value from converting the parcel to residential use, net the conversion costs, \( C_{i+1}(i,t) \), is greater than the expected value of keeping the land in an undeveloped state:

\[
\Pr[\{s(i,t) = 1 \mid s(i,t-1) = -1\} = \Pr[\{V_{i+1}(i,t) - C_{i+1}(i,t) \leq V_{i+1}(i,t)\}]
\]

The reverse probability of land being re-converted to an undeveloped state from a developed state is assumed to be highly improbable due to the likely economic
irreversibility of conversion. We treat it here as being a random shock and very small:  

\[
\text{Prob}\{s(i,t) = -1 \mid s(i,t-1) = 1\} = \omega \quad \text{where } \omega \ll 1
\]

**Interaction Effects**

The key dynamic feature of this model is the inclusion of a lagged interaction effect, \(I_s(i,t-1)\), which introduces the possibility of interdependence among developers’ residential conversion decisions over time. Interaction among agents is hypothesized to arise due to a variety of positive and negative spatial externalities generated from neighboring land use patterns. For example, positive spillover effects between residential developments may include various “community” spillover effects, e.g. people may find it desirable to live in close enough proximity to others as to feel part of a community. In addition, there may be positive effects associated with a critical density of residents in an area, which may be necessary to attract public and private services to the area. Negative spillover effects between neighboring development may arise from road congestion effects or a lack of privacy. Conversely, positive spillover effects may arise from undeveloped land in the form of open space amenities.

The term “spatial” externalities imply an explicit spatial relationship between the generator and receptor of the externalities. In this case, we assume that these externalities are a decreasing function of distance, but change at different rates over distance, depending

![Figure 1: Spatial Externalities as a Function of Distance](image)

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\[13\] This allows for the case of irreversible development, in which \(\omega = 0\).
on the nature of the externality (see Figure 1). The net externality effect – i.e. the net positive or negative effect of the surrounding land use pattern within a given radius of parcel i on parcel i’s value in either a residential or undeveloped use – will depend crucially on the extent of the neighborhood around parcel i that is considered. In the simple example illustrated in Figure 1, the net externality effect within the neighborhood determined by radius of length \(a\) is positive, whereas the net externality effect within the “doughnut” neighborhood surrounding this smaller neighborhood (delineated by points \(a\) and \(b\)) is negative. The overall net effect of these spillovers over the entire range of their influence (defined by distance \(b\) in Figure 1) will depend on their relative magnitudes.

In specifying the interaction effect generated by spatial externalities, we adopt a variant of the Ising model from statistical mechanics to describe land use interactions among developers. This model of pattern formation posits a “spin-spin” interaction among particles i and j as a function of distance between the two particles. Here, we draw the analogy between the spin of a particle and the land use of a parcel and therefore, the Ising model provides a straightforward means of capturing the interaction among parcels’ land use states as a function of distance. Given that only two land use states are possible, two types of interaction effects are specified, corresponding to the effect of externalities generated from neighboring land uses on the expected value of parcel i in a residential and undeveloped use respectively.\(^{14}\) A general expression for an Ising-type of interaction between the land use of parcel i in period t, \(s(i,t)\), and that of parcel i’s neighbor in period \(t-1\), \(s(j,t-1)\), can be written as (Bar Yam, 1997):

\[
I_s(i,t-1) = \sum_{j \in R} J_s(d_{ij})s(i,t)s(j,t-1)
\]

where \(J_s\) is an interaction parameter, which is a function of \(d_{ij}\), the distance between parcels i and j, and conditioned on the land use of parcel i, \(s = s(i,t)\). The summation is over the set of parcels within the neighborhood \(R\), where \(R\) is defined by radius \(a\) as a local neighborhood around parcel i. For example, if \(a = 1\), then \(R\) is the set of parcels within 1 unit of distance from parcel i or alternatively, the set of parcel i’s nearest neighbors. This specification of the interaction effect takes advantage of the \{-1,1\} indexing assigned to parcels in undeveloped and residential states respectively. Consider the expressions inside the summations: \(s(i,t)s(j,t-1)\). If the land use of parcels i and j are positively (negatively) correlated, then this term is positive (negative). Given that \(I_s(i,t-1)\) enters positively in the expected value function, \(V_s(i,t)\), the direction of the interaction parameter, i.e. \(J_s > 0\) or \(J_s < 0\), will determine whether positively or negatively correlated land use parcels increase the

\(^{14}\) In doing so, we continue to focus on the expected value of parcel i in a residential state, but clearly the opportunity cost associated with the conversion choice is also important and therefore, we model the expected value of parcel i in an undeveloped state. In doing so, the non-developed use is referred to as “undeveloped,” but we have in mind some productive use of the parcel in a non-developed state, e.g. agriculture or commercial forestry. Alternatively, this could be thought of in terms of a local government owning parcel i and therefore, \(V_s(i,t)\) would be interpreted as the expected social value of parcel i in an undeveloped state, e.g. as a recreational area or protected environmental area.
expected value of parcel \( i \) in land use \( s \).\(^{15} \) Based on this general expression, two interaction effects are specified, one for the value of parcel \( i \) in each of the two possible land uses.

First, consider the effect of surrounding land use pattern within a given neighborhood of parcel \( i \), defined as \( R^s_{t+1}(i) \),\(^{16} \) on the expected residential value of parcel \( i \). We surmise that neighboring residential parcels generate both positive and negative externalities that may impact the value of parcel \( i \) in a residential use and that the direction of these spillover effects is determined by the relative distance between parcel \( i \) and neighboring residential parcel \( j \). In addition to the negative externalities from developed parcels within this neighborhood, we allow for positive spillover effects from open space, which are increasing in the amount of undeveloped parcels within both neighborhoods. Taken together, the interaction effect for the expected residential value of parcel \( i \) in period \( t \) is defined as:

\[
I_1(i,t-1) = \sum_{j \in (R^1_t \cup \{i\}) \cup \{j \}} J_{11}(d_{ij}) s(i,t) s(j,t-1) + \sum_{j \in (R^1_t \cup \{i\}) \cup \{j \}} J_{121}(d_{ij}) s(i,t) s(j,t-1) + \sum_{k \in R^1_t} J_{21}(d_{ik}) s(i,t) s(k,t-1)
\]

where \( J_{11} > 0 \) and \( J_{21} < 0 \).\(^{17} \) The first summation is over developed parcels only within neighborhood \( R^1_t \) and captures the positive development spillovers from existing residential development. The second term is over undeveloped parcels only within this same neighborhood \( R^1_t \) and captures the positive spillover effects from open space in this neighborhood. The third term is over both developed and undeveloped parcels and captures the concurrent negative development externalities and positive open space effects from the neighborhood \( R^2_t \).

In constructing the interaction effect for parcel \( i \)'s expected value in an undeveloped state, we hypothesize that other undeveloped parcels within parcel \( i \)'s neighborhood increase the expected value of parcel \( i \) in an undeveloped state and vice versa, that developed parcels within this neighborhood decrease parcel \( i \)'s value in an undeveloped state. For example, suppose the undeveloped use is agriculture. The presence of other agricultural lands around parcel \( i \) may increase the value of parcel \( i \) in an agricultural use because they contribute to the viability of the area as an agricultural area and therefore, services and other infrastructure that support agriculture may be more likely to be located within close proximity of parcel \( i \). On the other hand, the fragmentation of the surrounding landscape with residential development may decrease the agricultural

\(^{15} \) Note that in this most general formulation of the interaction effect, there will always be “concurrent” positive and negative externality effects from any given set of parcels within a neighborhood: one effect generated from the set of developed parcels and the opposite effect from the set of undeveloped parcels.

\(^{16} \) The superscript indexes the particular neighborhood, as defined by the area encompassed by a minimum and maximum distance from parcel \( i \). The subscript indexes the expected land use of parcel \( i \) in period \( t+1 \). This dependence on parcel \( i \)'s land use reflects the notion that the spatial extent of different externalities, which are conditional on whether parcel \( i \) is in a residential or undeveloped state, are not necessarily equal. Note that neighborhoods are assumed to be non-overlapping and therefore, each neighborhood is a doughnut-like shape. Only the nearest neighborhood is contiguous to parcel \( i \).

\(^{17} \) In what follows, we sometimes refer to \( J_{11} \) term as a positive interaction and the \( J_{21} \) term as a negative interaction effect.
value of parcel i for several reasons, e.g. increased congestion on the roads from non-farm residents and the loss of an agricultural-based economy in the region.\(^\text{18}\) Given these considerations, the interaction effect for the value of parcel i in an undeveloped use is specified as:

\[
I_{-1}(i, t - 1) = \sum_{j \in R_{-1}} J_{-1}(d_j) s(i, t)(s(j, t - 1)
\]

where \(J_{1-1} > 0\) and \(J_{2-1} = 0\). This specification allows for simultaneous positive spillovers from neighboring undeveloped land and negative externalities from neighboring residential parcels within the neighborhood defined by \(R_{-1}(i)\). By setting \(J_{2-1} = 0\), we rule out the possibility of negative externalities from neighboring undeveloped land and positive spillovers from residential parcels.

### 5. Spatial Implications Of Interaction Among Developers

The remainder of the paper focuses on the spatial implications of the conversion of land from an undeveloped to residential state, given the model of land use conversion specified above. In particular, an understanding is sought of the conditions under which a sprawl pattern of development is most likely to emerge. Previously we likened sprawl development to a negatively correlated land use pattern, in which residential subdivisions are negatively correlated and interspersed with undeveloped land (see Map 2). Using this notion of sprawl development, we identify the conditions under which negative correlation among residential parcels is most likely. Negative spatial correlation is defined in terms of the model as the negative correlation between parcel i’s expected land use choice and the existing amount of developed land within parcel i’s neighborhood. A high probability of a negatively correlated land use is predicted if: (1) the conversion probability of parcel i is high and the amount of neighborhood development is low and (2) the conversion probability of parcel i is low and the amount of neighborhood development is high.

In what follows, an analytical expression is derived for the expected land use of parcel i as a function of the spatial externality effects from neighboring land uses. We use this equation to describe the relative influence of interaction effects on the resulting correlation of land uses between parcel i and a set of neighboring parcels. While it is straightforward to construct an expression for the joint probability of all N parcels from this equation, solving the model for an equilibrium land use pattern is not possible so long as we assume an exogenous growth pressure effect.\(^\text{19}\) In this case, the region is seen as open to population and income increases that increase the regional demand for residential housing. These exogenous flows of population and other inputs can be viewed as

\(^{18}\) Alternatively, if the undeveloped state is a natural state, e.g. a noncommercial forest, and the agent maximizing parcel i’s expected value in this undeveloped state is a social planner, it is reasonable to assume that the same types of externalities may be present. For ecological reasons, the value of parcel i in a natural state is increasing in the amount of surrounding undeveloped land, which act as a buffer to development, and decreasing in the amount of residential parcels, which create a host of negative externalities, e.g. pollutant run-off and fragmentation of habitat.

\(^{19}\) It may be possible to characterize the equilibrium solution for a fixed level of population using analytical results from results in the statistical mechanics literature. This is something that I am currently working on.
“constraints” that keep the system out of equilibrium. In such a case, it is the dynamics of the system, rather than the steady state behavior, that is of interest.\textsuperscript{20} A simulation approach is used to further investigate the conditions under which a sprawl development pattern is predicted to evolve. In particular, we determine whether the conditions that yield a high probability of negative correlation between parcel \(i\) and the neighboring parcels are also predicted to lead to a sprawl pattern of development over a longer time horizon and larger spatial extent.

We first define an analytical expression for the conversion probability of parcel \(i\) in period \(t\), which is conditional on the lagged parcel characteristics, \(H_s(i,t-1)\), and the lagged land use states of neighboring parcels in period \(t\), captured by the interaction effect \(I_s(i,t-1)\). Several assumptions about the error terms are made: (1) the error component of the expected value function is assumed to be additive, so that \(V_s(i,t) = W_s(i,t-1) + \varepsilon_s(i,t)\), where \(W_s(i,t-1)\) is the deterministic portion of the expected value function; (2) the errors \(\varepsilon_1(i,t)\) and \(\varepsilon_{-1}(i,t)\) are assumed to be independent and (3) the errors are assumed to be extreme-value distributed. Furthermore, we assume that the functional form of the deterministic portion of the value function is additive and therefore: \(W_s(i,t) = H_s(i,t) + I_s(i,t)\). Lastly, we simplify the notation by (1) assuming that the conversion costs, \(C_{1|-1}(i,t)\), are determined by parcel characteristics, e.g. slope, soil type, and vegetative cover of parcel \(i\) in period \(t-1\), and are therefore captured in \(H_1(i,t-1)\) and (2) moving up the conversion decision to period \(t+1\), so that the spatial distribution of characteristics are written as a function of \(t\).\textsuperscript{21} The conditional probability that parcel \(i\) is converted, which we will also refer to as the expected land use of parcel \(i\) in period \(t+1\), \(s(i,t+1)\), can now be written as:

\[
(8) \quad s^e(i,t+1) = \frac{\exp\{H_s(i,t) + I_s(i,t)\}}{\exp\{H_s(i,t) + I_s(i,t)\} + \exp\{H_{-1}(i,t) + I_{-1}(i,t)\}}
\]

To simplify the following exposition, we assume that the net externality effect is not a continuous function of distance, but rather that it differs discretely among neighborhoods, \(R_d^d\), and is constant within each of the neighborhoods. The analysis is further simplified by assuming that all externality effects extend over the same neighborhood and therefore \(R_1(i) = R_2(i) = R_3(i) = R(i)\).\textsuperscript{22} Given that \(J_1 > 0\) and \(J_2 < 0\), this implies that the net externality effect is either positive or negative over the entire range of influence, \(R\), depending on the relative magnitudes of the interaction parameters. The interaction effects are now written as:

\textsuperscript{20} For example, if conversion is fully irreversible, then the state in which all parcels are developed is an absorbing state and therefore, a steady state solution. However, this is clearly an uninteresting state, given that we are interested in the evolution of the land use pattern from an initially almost fully undeveloped state.

\textsuperscript{21} The only purpose of this is to shorten the notation, so that the lagged functions are written as a function of \(t\) rather than \(t-1\).

\textsuperscript{22} This assumption does not change the qualitative results for the analysis in this section. However, when the evolution of land use pattern is considered over an entire region comprised of multiple parcels and over multiple time periods, the relative neighborhood extents of various spatial externalities is found to be a critical determinant of land use pattern. Therefore, the distinction between neighborhood extents is reintroduced in Section 4.4.
where \( s(i,t+1) \) is dropped from the summations, as \( s(i,t+1) = 1 \) in Equation (9) and \( s(i,t+1) = -1 \) in Equation (10), which is reflected by the addition of a negative sign. The effects of these interactions on parcel \( i \)'s conversion probability are considered for two cases: (1) parcels are spatially homogeneous other than their land use and therefore, all conversions occur as a result of the interaction effects and (2) parcels are heterogeneous due to both their land use and exogenous features of the landscape, which positively influence the parcel's conversion probability and therefore, may either offset or reinforce the net interaction effects.

**Case 1: Endogenous Interaction Effects**

For Case (1), in which all changes in the land use pattern are due to the interaction effects associated with parcel \( i \) in either a residential or undeveloped use and a random component, the expected land use of parcel \( i \) in period \( t+1 \) is:

\[
s^E(i,t+1) = \frac{\exp[I_1(i,t)]}{\exp[I_1(i,t)] + \exp[I_{-1}(i,t)]}
\]

where \( I_1(i,t) \) and \( I_{-1}(i,t) \) are defined as in Equations (9) and (10).

Figures 2A-F illustrate how the probability of conversion varies over the density of development, \( D \), within the neighborhood defined by \( R \) for varying interaction parameter values. From these figures, we see that the probability of conversion varies over both the density of surrounding development and the relative magnitudes of the interaction parameters, \( J_{21} \), \( J_{11} \) and \( J_{1-1} \). In particular, we note that the case in which negative correlation is most likely, exhibited in Figure 2C, is characterized by a relatively large value of the \( J_{21} \) parameter in comparison to the other interaction parameters. This makes intuitive sense, given that the \( J_{21} \) term reflects both the positive spillover effects from open space and the negative externalities from development on the residential value of parcel \( i \). Therefore, very low levels of surrounding development create strong positive spillover effects from open space. This effect competes with the opportunity cost of conversion,

\[^{23}\text{Here we have in mind exogenous characteristics that contribute more to the value of parcel } i \text{ in a residential use than its value in an undeveloped state, e.g. distance from roads, cities, or the provision of public water and sewer services, and therefore limit attention to the case in which } H_1 > H_{-1} \geq 0.\]

\[^{24}\text{Because we assume a regularly sized grid, the area assigned to each parcel is equal. This, along with the constraint that there is only one level of density at which parcels may be developed, implies that the number of developed parcels in a neighborhood is the same as the density of development within the neighborhood, where density is defined as the cumulative developed area as a fraction of the total area of the neighborhood. Therefore, only two land use states are possible, this variable also defines the amount of open space in the neighborhood.}\]
captured by the $J_{1-1}$ term, which is also a function of the surrounding landscape and also increases with the amount of undeveloped land in the neighborhood. For a sufficiently large value of $J_{2}$ relative to $J_{1}$, the expected returns from conversion outweigh the opportunity costs and the probability of conversion is high for low levels of neighboring development. The positive spillover effect from surrounding development, captured in the $J_1$ term, reinforces this effect. However, as the amount of development in the neighborhood increases, both the positive effect from neighboring development and the negative externalities from this development grow stronger. For a sufficiently dense level of neighborhood development, the negative externality effects from development dominate the positive development externalities and the probability of conversion approaches 0.

Other figures show how varying values of the interaction parameters influence the negatively correlated land use pattern exhibited in Figure 2C. From Figure 2A, we see that larger values of $J_{1-1}$ will overwhelm the negative correlation result and, for a sufficiently strong $J_{1}$ parameter, the opposite case from sprawl development, i.e. positive correlation among land uses, results. In this case, the opportunity costs are also a function of the land use pattern such that, the positive spillover effects from undeveloped land increase the value of parcel $i$ in an undeveloped use more than in a residential use and therefore, the conversion probability is low for low levels of $D$. Vice versa, for largely developed neighborhoods, the value of parcel $i$ in an undeveloped state is relatively low, and therefore, the probability of conversion is high, ceteris paribus.

Increases in the $J_{1}$ parameter also overwhelms the negative correlation result, but has a different effect on the probability function. The parameter $J_{1}$ captures the relative value of positive spillover effects from neighboring residential parcels on the residential value of parcel $i$. Therefore, if $D > 0$, $J_{1} > 0$ will always increase the relative value of conversion. Figures 2E-F show that as $J_{1} > 0$ increases relative to the other parameters, the positive effects from neighboring development diminish the effects of the negative development externalities and the probability of conversion approaches 1 for increasingly higher levels of surrounding development. For a sufficiently large $J_{1}$ parameter, the conversion probability approaches 1 for all levels of neighboring development.\footnote{This effect is a type of “agglomeration” effect, which is the typical type of interaction among firms cited by the urban economics literature to explain the evolution of cities.}

**Case 2: Exogenous Factors and Endogenous Interaction Effects**

In the following case, we allow for the consideration of exogenous factors, specific to an individual parcel, in the calculation of the expected land use choice:

\begin{equation}
\begin{aligned}
s^e(i, t + 1) &= \frac{\exp{\{\alpha_1 h_i(i, t) + I_1(i, t)\}}}{\exp{\{\alpha_1 h_i(i, t) + I_1(i, t)\}} + \exp{\{\alpha_{-1} h_{-1}(i, t) + I_{-1}(i, t)\}}}
\end{aligned}
\end{equation}

Figures 3A-D illustrate the influence of exogenous features on the conversion probability. In general, the inclusion of $\alpha'_s h_s$ in the value term acts to shift $s^e(i, t+1)$ to the left or right, depending on the relative magnitudes of $\alpha_i$ and $\alpha_{-1}$ and on the relative
magnitudes of the interaction parameters. Here we are interested in the case in which
which \( \alpha = \alpha_1 - \alpha_{-1} > 0 \). For \( \alpha > 0 \), the probability function is shifted such that the range of
neighborhood development over which the probability of conversion approaches 1
increases (Figure 3B). Intuitively, as exogenous factors contribute more to the value of
parcel \( i \) in a residential state relative to their contribution to the value of parcel \( i \) in an
undeveloped state, this serves to shift the expected land use choice in favor of the
residential state. In Figure 3B the contribution to the value of the parcel in a developed
state from the exogenous factors competes with the negative interaction effect from the
surrounding development and increases the range of \( D \) for which the conversion
probability of parcel \( i \) approaches 1. For an intermediate range of \( \alpha > 0 \), the conversion
probability will still be low for a sufficiently large \( D \) due to the negative development
externalities. However, if \( \alpha \) is sufficiently large relative to \( J_2 \), the probability of
conversion will approach 1 for all levels of neighborhood development. Figures 3C-D
show how the relative influence of a fixed value of \( \alpha \) changes as the magnitude of \( J_2 \)
increases. It is clear that as the relative magnitude of the interaction parameter \( J_2 \)
increases, the influence of the exogenous factors diminishes. For a sufficiently large value
of \( J_2 \), the influence of the exogenous factors disappears.

Based on the above observations, we conclude that negative correlation between
parcel \( i \)'s expected land use and the land use of surrounding parcels is dependent on a
minimum critical magnitude of the \( J_2 \) parameter relative to the other parameters. In
particular, the magnitude of \( J_2 \) must be strictly greater than \( J_{1,-1} \), which exactly offsets the
influence of \( J_2 \). In addition, for any value of \( J_{1,1} > 0 \), \( J_2 \) must be sufficiently greater than
both \( J_{1,-1} \) and \( J_{1,1} \) in order to offset the reinforcing effects on the conversion probability
from \( J_{1,1} \) when \( D \) is large (compare Figures 2C, 2G, and 2H). Because both \( J_{1,1} > 0 \) and \( J_{1,-1} > 0 \) will increase the probability of conversion when the amount of surrounding
development is high, the offsetting effects from \( J_2 \) must be greater than these combined
effects to maintain a pattern of negative correlation (i.e. a low probability of conversion,
given a large amount of surrounding development). The same is true for values of \( \alpha > 0 \).
In this case, the \( J_2 \) parameter must be sufficiently large to offset the positive influence of
these exogenous features on the probability of conversion. Otherwise, because the positive
effects from the exogenous factors raise the conversion probability for all levels of \( D \), the
pattern of negative correlation is not maintained for cases in which the amount of
neighboring development is large. We conclude that a negatively correlated pattern of land
use between parcel \( i \) and parcel \( i \)'s neighbors will result if the following conditions hold:

(1) \[ |J_2| > J_{1,-1} \text{ and } J_{1,1} = \alpha = 0 \] or

(2) \[ |J_2| > J_2^* \]

where \( J_2^* = f(J_{1,-1}, J_{1,1}, \alpha) \) is the minimum critical value (in absolute value terms) of \( J_2 \)
relative to the other parameters, \( J_{1,-1} \geq 0 \), \( J_{1,1} \geq 0 \), and \( \alpha \geq 0 \), necessary to offset their
combined effects such that the probability of conversion approaches 0 for a large amount
of surrounding development. For example, in Figure 2C, \( J_2 = 3 \), given \( J_{1,-1} = J_{1,1} = 1 \) and
\( \alpha = 0 \).
In addition, we note that several instances arise in which a high probability of conversion will occur with a fully undeveloped neighborhood as the result of influences other than the spatial externalities captured by the $J_2$ parameter. For example, in the case of a relatively large $\alpha > 0$ effect, the probability of conversion is high for all values of $D$ and therefore, this effect will cause a negatively correlated land use pattern between parcel $i$ and its neighbors when $D$ is very small or $D = 0$. In addition, a negatively correlated land use pattern may occur as the result of random effects, which are most likely to dominate when the interaction effects and exogenous factors offset each other such that the value of conversion net opportunity costs is close to zero. Borrowing from Manski (1993), we distinguish between these exogenous effects and the endogenous spatial externalities, captured in the $J_2$ term, both of which may lead to negative correlation among land uses. Exogenous effects, both observed and unobserved, create negative correlation as a result of positive correlation between residential development and exogenous factors that are spatially distributed such that a negatively correlated land use pattern results. For example, hilltop views are negatively correlated in space and may be positively correlated with residential development and therefore, ceteris paribus, a negatively correlated development pattern would emerge. In contrast, negative correlation of land use pattern due to endogenous effects arises precisely because of the spatial externalities from neighboring land uses. While these two different effects may generate an observationally equivalent land use pattern, the distinction has important implications for the evolution of land use pattern over time.

**Evolution of a Regional Land Use Pattern**

In order to understand the consequences of interaction effects over a longer time horizon and larger spatial extent, the evolution of an aggregate land use pattern is simulated as the cumulative result of many individual profit-maximizing decisions made by developers at the parcel level. By “aggregate” land use pattern we mean the pattern of residential land use conversion over a relatively large region within an exurban area, which is comprised of many land parcels owned by many individual developers. Based on the model in the previous section, developers’ conversion decisions are represented in probabilistic terms and therefore, we need a means of translating conversion probabilities into actual land use states. The total amount of conversion in any given time period is clearly a function of regional growth pressures, e.g. population and income changes. Here we simplify the regional growth effects by assuming that the regional demand for new housing is constant such that one new conversion occurs in each period. Given this, we assume that the parcel with the highest probability is the parcel that is converted. Once converted, the probability of a parcel’s re-conversion to an undeveloped state is assumed to be very close to zero.

A type of cellular automaton is used to simulate the stochastic evolution of a residential land use pattern. We assume that land use parcels are distributed on a 30x30

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26 The difference between this type of simulation and the more standard cellular automata is the lack of simultaneity in the evolution process. This arises because (1) the lagged effect among agents making land use decisions and (2) exogenous factors and growth pressures in the region, which also influence the state of a parcel.
grid of equal area cells. The residential land use pattern is first simulated for different interaction parameter values, which correspond to the magnitude of spatial externalities generated by the neighboring land uses, and the influence of observed exogenous features is ignored. In a second round of simulations, we allow for consideration of an exogenously located city center and road. In all cases, the initial conditions are specified by an almost fully undeveloped region, in which one parcel is initially in a residential state. In order to compare the outcomes of the simulations, we start with the same, randomly determined initial state. Given that one conversion occurs in each period and that once converted, a parcel is rarely unconverted, the simulation reaches a “natural” end point after 900 time steps (i.e. the entire region is developed). Because the interest here is in identifying the qualitative pattern that emerges for different parameter values, it is unnecessary to simulate to this final end state. We find via trial and error that approximately 100 time steps are sufficient for this purpose. Multiple simulations were performed for each of the following two cases:

**Case 1**: Interaction effects only: \( J_{11} > 0, J_{21} < 0, J_{11} > 0, \alpha_s = 0 \). The value of each parcel in residential and undeveloped uses is determined only by positive and negative effects associated with neighboring land uses. All other landscape heterogeneity is ignored and therefore, the evolution of a land use pattern is fully endogenous. The neighborhood for each agent is defined using inverse distance weights so that nearest neighbor interactions have the greatest effect on the parcel’s residential value. The neighborhood is locally defined, so that beyond a certain cut-off distance, development does not influence a parcel’s value.

**Case 2**: Interaction effects with a city center and road: \( J_{11} > 0, J_{21} < 0, J_{11} > 0, \alpha_1 > 0, \alpha_2 > 0 \). Two exogenously determined features – a city center and a road that cuts through the region to the city center – differentiate the region. Each parcel’s value in residential and undeveloped uses is an additive function of these two exogenously determined effects and the interaction effects from development. We assume that the location of both the road and the city increases parcel i’s residential value more than the undeveloped value.

Figures 4-6 illustrate results from the simulations of land use pattern for varying values of the interaction parameters for Case 1, i.e. no additional heterogeneity of the landscape other than the land use pattern itself, and \( R_{11}^1 < R_{21}^2, R_{11}^1 + R_{21}^2 = R_{-1} \). Figure 4 shows the evolution of a residential pattern for the same minimum value of \( J_{21} \) (relative to \( J_{11} \) and \( J_{11} \)) found in the previous analysis to generate a high probability of negatively correlated land uses (shown in Figure 2C). In comparison, Figure 5 shows the evolution of a pattern for a relatively stronger \( J_{21} \) parameter (analogous to Figure 2D). In both cases, the evolution begins in a very scattered manner due to the dominating influence of the negative interaction parameter, \( J_{21} \). However, once a certain density of development is reached in the region, the two patterns diverge. New development in Figure 4 begins to

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27 In words, the neighborhood for which net positive development externalities exist is smaller than the neighborhood for which net negative development externalities exist. In addition, the total neighborhood extent of externalities that influence the value of the parcel in both residential and undeveloped uses is the same. While simulations with different relative neighborhood extents were also performed, this set-up yielded the most interesting and relevant results.
form around existing development, such that small clusters of multiple residential parcels emerge. In contrast, the development pattern in Figure 5 maintains a negative correlation among developed parcels. Comparison of the two patterns after 75 periods shows that two qualitatively different patterns emerge as a result of the variation in the $J_2$ parameter. The difference can be understood in terms of the competing effects among the various externalities. In Figure 4, the average value of converting a parcel to residential use decreases as the density of development in the region increases, due to negative development externalities. At the same time, the relative influence of the positive development externalities, which raises the value of developing a parcel contiguous to one or more developed parcels, increases with overall development density. Therefore, for a sufficient level of development in the region, the positive externalities from development overtake the negative externalities and induce clustering. This tendency is reinforced by the opportunity costs, which are lowest when a parcel is surrounded by development. However, for a sufficiently strong $J_2$ parameter (Figure 5), these competing effects never overtake the negative interaction effect and the noncontiguous development pattern is maintained for higher levels of regional development density.

Figure 6 illustrates the evolution of pattern for different levels of increase in the $J_1$ parameter, which determines the opportunity costs that are also as a function of land use pattern. For a large increase in $J_1$, shown in the top three snapshots in Figure 6A, the evolution of pattern is fully contiguous. This reflects the relatively high value of parcels surrounded by undeveloped parcels in an undeveloped use, which prevents these parcels from being converted. Conversely, parcels with development in their neighborhood have both a lower value in the undeveloped use and a higher value in residential use, due to positive development externalities from the $J_1$ term. Although we don’t show it here, the same pattern emerges for a sufficiently large value of $J_1$ relative to the other parameters. The bottom three snapshots in Figure 6B show the evolution of pattern for a small increase in $J_1$. In this case, the formation of clusters begins right away, due to the influence of $J_1$. But, because the negative externalities from development are still relatively strong, cluster size remains small and the distance between clusters is larger. In comparing Figures 4 and 6B, we note that both patterns are characterized by cluster formation. This suggests that a clustering pattern will emerge for an intermediate range of $J_2$, relative to $J_1$ and $J_1$, in which a “tension” is created by the competing externality effects. This tension results in one or the other effect dominating, depending on the level of neighborhood development, and therefore both scattered and contiguous development occur, which results in clustering. If the relative values of the interaction parameters do not fall within this intermediate range, then either a fully non-contiguous pattern (Figure 5) or contiguous pattern (Figure 6A) emerges.

Figures 7-10 illustrate Case 2, in which the influence of an exogenously defined city center and road is considered. We find that depending on the value of $J_2$ relative to other parameters, one of three general patterns will emerge: (1) for a very large value of $J_2$ relative to the other parameters, a fully scattered pattern will emerge; (2) for a wide range of intermediate values of $J_2$ relative to the other parameters, different types of

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28 This is similar to Krugman’s (1996) result, in which the tension between “centrifugal” and “centripetal” forces causes clustering of firms.
clustered patterns will emerge, which vary according to the placement of the clusters (which are influenced by the magnitudes of the exogenous parameters), cluster size, and the distance between clusters; and (3) for small values of $J_2$ relative to the other parameters, a fully contiguous pattern emerges, as determined by the exogenous features.

Of particular interest is Figure 9, in which the influence of a road and a city center create positive correlation among residential parcels and therefore compete with the negative interaction effect from positive open space effects and negative development externalities. The location of development is clearly determined by the exogenous landscape features, but the repelling effect from negative externalities causes development to leapfrog over several parcels away from the city center, to an undeveloped area that is adjacent to the road. Once the areas adjacent to the road have reached a certain level of noncontiguous development, further development once again leapfrogs away from existing development -- this time to an undeveloped area that is nonadjacent to the road, but closer to the city center.

In contrast, Figure 10 shows the same evolution, but for larger values of $J_1$ and $J_1$, both of which account for spatial externalities that offset the negative interaction effect. In this case, larger clusters form around the road and eventually, all parcels that are directly adjacent to either the road or the central city are developed. Only at this point does development jump away from these exogenous features to an undeveloped area. For a very weak negative interaction effect, in which the exogenous effects and other interaction parameters dominate, there is no leapfrog effect and a fully contiguous pattern emerges (see Figure 8).

Comparison of Figure 9 to the observed pattern of residential subdivisions in Map 2 shows a qualitative similarity between the two patterns. In both cases, the residential parcels are located in clusters, some of which are adjacent to the road. In the simulation, this pattern was generated via a mix of endogenous and exogenous effects: (1) a positive effect from proximity to the road and (2) a strong negative interaction effect and weak positive interaction effect, both created by spatial externalities from neighboring parcels.

Conclusion

Results from the simulations confirm the initial hypothesis regarding the necessary conditions for a non-contiguous sprawl development pattern to emerge as a function of endogenous and exogenous factors. Namely, for a sufficiently strong negative interaction parameter, $J_2 < |J_2|$, a negatively correlated pattern of residential land use pattern will evolve. However, the simulations also reveal that for an intermediate range of $J_2$ below this threshold value, $J_2 < |J_2| < J_2*$ the resulting pattern is characterized by clusters of developed parcels. We find that this result occurs for a range of parameter values, in which the key parameter value is again the negative interaction effect. For a sufficiently low value of $J_2$ relative to the other parameters, $|J_2| < J_2*$, the evolution of development is fully contiguous and will either (1) agglomerate around an initial developed parcel (Figure 6A) or (2) will occur in a contiguous pattern around the exogenous features (Figure 8). We conclude that a sufficiently strong negative interaction
effect, i.e. $J_2^{**} < |J_2|$, is a viable hypothesis for the explanation of the observed pattern of non-contiguous residential development.

Of course, negative correlation may arise from other features of the landscape that are negatively correlated in space and which influenced the residential value of parcels. However, most features of the landscape that influence the residential value of a parcel are not scattered across the region like hilltops, but rather are either concentrated in one or several areas -- e.g. a city or town, shopping area, public services, recreational site -- or are located continuously along the edges of parcels, e.g. a road or waterfront. In either case, these exogenous features would induce positive correlation among developed parcels. While the possibility of exogenously determined negative correlation has certainly not been ruled out here, there seems to be a lesser number of competing hypotheses to explain a negatively (vs. positively) correlated pattern, which strengthens the case for endogenous effects that lead to negative correlation among residential parcels.

The presence of endogenous interaction effects has implications for the evolution of land use pattern over time. Namely, these effects generate a “self-organizing” dynamic, in which the particular pattern that evolves is both a function of past spatial externalities and a generator of future spatial externalities, which in turn influence future land use pattern. This influence of past land use pattern on current and future land use pattern via dynamic spatial externalities implies a path dependent process of land use conversion. Given the stochastic nature of land use conversion, the presence of these endogenous effects implies that randomness, or what Arthur (1988a) calls “historical chance,” may play a defining role in the evolution of a land use pattern. For example, a random event which leads to the conversion of a parcel may influence the regional evolution of land use pattern via spatial externalities -- e.g. given both positive and negative externalities from development, it could lead to a clustering of development around this initially randomly converted parcel. In contrast, a pattern generated by exogenous features will always evolve towards the same pattern, so long as the exogenous influences remain the same. In this case, the evolution is path independent and therefore, random events are unimportant in determining the resulting spatial pattern. The presence of endogenous effects has implications for growth management policy. Under the right conditions, small scale changes in the landscape may be “magnified” via dynamic spatial externalities and lead to large scale changes in land use pattern at a regional level. These regional consequences of localized changes underscore the importance of a regional approach to land use management.

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29 For example, the formation of Silicon Valley has been explained as a consequence of the combination of historical chance and positive spillover effects among firms (Arthur, 1988a).
References


Figures 2A-F: Case 1
Probability of Conversion for Different Values of $J_2$, $J_{1-1}$, and $J_1$

**Figure 2A:** $J_2 = 0$, $J_{1-1} = 1$, $J_1 = 1$

**Figure 2B:** $J_2 = -1.5$, $J_{1-1} = 1$, $J_1 = 1$

**Figure 2C:** $J_2 = -3$, $J_{1-1} = 1$, $J_1 = 1$

**Figure 2D:** $J_2 = -10$, $J_{1-1} = 1$, $J_1 = 1$

**Figure 2E:** $J_2 = -2$, $J_{1-1} = 1$, $J_1 = 1$

**Figure 2F:** $J_2 = -2$, $J_{1-1} = 1$, $J_1 = 2$
Figures 2G-H: Case 1 (cont.)
Probability of Conversion for Different Values of $J_{21}$, $J_{1.1}$, and $J_1$

Figure 2G: $J_{21} = -1, J_{1.1} = J_1 = 0$

Figure 2H: $J_{21} = -4, J_{1.1} = 2, J_1 = 1$

Figures 3A-D: Case 2
Probability of Conversion for Different Values of $\alpha$ and $J_{21}$, Given $J_{1.1} = J_1 = 1$

Figure 3A: $\alpha = 0, J_{21} = -3$

Figure 3B: $\alpha = 2, J_{21} = -3$

Figure 3C: $\alpha = 2, J_{21} = -6$

Figure 3D: $\alpha = 2, J_{21} = -8$
Case 1: Endogenous Effects Only

Parameter Values: \( J1(2) = -3, J1(-1) = 1, J1(1) = 1, \alpha = 0 \)
Figure 5

Case 1: Endogenous Effects Only
Parameter Values: J1(2) = -6, J1(-1) = 1, J1(1) = 1, alpha = 0

Initial conditions

Period 5

Period 15

Period 50

Period 60

Period 75
Case 1: Endogenous Effects Only

Figure 6A
Parameter Values: J1(2) = -2, J1(-1) = 2, J1(1) = 1, alpha = 0

Initial conditions
Period 10
Period 50

Figure 6B
Parameter Values: J1(2) = -3, J1(-1) = 1.2, J1(1) = 1, alpha = 0

Period 5
Period 25
Period 50
Figure 7

Case 2: Exogenous and Endogenous Variables

Parameters: J1(1) = J1(-1) = 1, J2(1) = -3, alpha1 = 2, alpha2 = 0

Initial conditions

Period 5

Period 10

Period 50
Figure 8

Case 2: Exogenous and Endogenous Variables
Parameters: $J1(1) = J1(-1) = J2(1) = 0$, $\alpha1 = 2$, $\alpha2 = 0$
Case 2: Exogenous and Endogenous Variables

Parameters: J1(1) = 1, J1(-1) = 0, J2(1) = -3, alpha1 = 1 alpha2 = 2

Initial conditions

Period 5

Period 15

Period 65
Case 2: Exogenous and Endogenous Variables
Parameters: \( J1(1) = J1(-1) = 1, J2(1) = -3, \alpha_1 = 1, \alpha_2 = 1 \)

Figure 10
recent residential subdivisions
Calvert County