Stock Externality Regulation Under Uncertainty

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Working Paper

June 19, 1998
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Abstract

Uncertainty about costs causes otherwise equivalent price and quantity controls to result in different quantity responses and costs. Weitzman (1974) found that the efficiency of price relative to quantity controls in a regulated market depends critically on both the existence of cost uncertainty and the relative slopes of marginal benefits and costs. We explore how uncertainty could influence the choice of policies to regulate flows when benefits are associated with the stock, rather than the flow. For example, in controlling emissions of greenhouse gases, the stock of greenhouse gases is associated with global climate change. Other environmental and natural resource examples include policies that address ozone-depleting substances, land use change, species preservation, hazardous waste disposal, natural resource conservation, and groundwater pollution. Potential non-environmental examples include research and development, educational attainment, and monetary policy.

Using a simple analytical model that incorporates costs and benefits, stock decay, time discounting, and uncertainty, we uncover five important principles governing the choice of price-based policies (e.g., taxes) relative to quantity-based policies (e.g., tradeable permits) for controlling stock externalities. First, the slower a stock decays (or the longer the stock persists), the more likely it is that quantity controls are preferred to price controls because deviations in flows under a price policy persist over long periods of time. Second, low discount rates give greater weight to these future deviations and thus reinforce the policy divergence over time, thereby favoring quantity controls. Third, in a multi-period analysis results hinge on correlation across periods. In particular, positive correlation among cost shocks across time tend to decrease the net benefits of using prices, thereby favoring quantities. Fourth, controlling for the aforementioned effects, the relative slopes of the marginal benefits and costs of controlling the externality continue to be critical determinants of the efficiency of prices relative to quantities, with flatter marginal benefits and steeper marginal costs favoring prices. Fifth, and finally, we find that the assumption of a globally quadratic benefit function almost guarantees that price controls will be more efficient than quantity-based policies for controlling stock externalities. In the case of climate change, these results suggest that the current focus on quantity-based policies, to the exclusion of policies that include price-like elements, may be inappropriate.

Key words: Stock, Externality, Regulation, Policy, Uncertainty, Price, Quantity, Tax, Tradeable Permit, Pollution, Climate Change

JEL Classification No(s): Q28, D81, C68
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1. Introduction

In a deterministic world, it is widely recognized that regulation based on either prices (e.g., taxes or subsidies) or quantities (e.g., tradeable permits) can yield any desired level of output, including the economically efficient outcome. Although state-contingent policies could, in principle, be designed to maintain this proposition even under conditions of uncertainty, such policies would be of little if any practical use. Recognizing this, Weitzman (1974) initiated a discussion about the relative efficiency of alternative regulatory instruments in a distinctly different world characterized by uncertainty, asymmetry of information, second-best policy alternatives, and costly policy adjustments. Under these more realistic conditions, Weitzman found that there are fundamental differences in the consequences of price versus quantity regulation, and he described the resulting divergence in efficiency as a function of fairly basic and intuitive economic variables, including the slopes of marginal benefits and costs and the degree of cost uncertainty.

With the possible exception of certain research on fisheries\textsuperscript{1} and globate climate change\textsuperscript{2}, however, virtually all subsequent work on optimal policy choice under uncertainty has dealt with a situation where both benefits and costs are a function only of current production, as in Weitzman’s original formulation.\textsuperscript{3} In contrast, we explore how uncertainty could influence the

\textsuperscript{*} Newell and Pizer are Fellows at Resources for the Future. The paper has benefited from helpful comments from participants at an RFF seminar and support under NSF Grant #9711607.

\textsuperscript{1} Research on efficient fisheries management has addressed the issue in a context where benefits and costs are a function of the stock of fish, via the harvesting function (Koenig 1984a, 1984b; Anderson 1986; Androkovich and Stollery 1991). However, parsimonious, intuitive results regarding efficient policy for controlling other types of stocks are obscured in the analyses by the particulars of optimal fishery modeling.

\textsuperscript{2} Economic analyses of carbon abatement policy have developed the clearest intuition about the importance of uncertainty for policy instrument choice in a stock setting, but this has not been a focus of that research, which has also been burdened by the complexities of modeling the benefits and costs of climate change mitigation. While several authors document the relative flatness of the marginal benefits of CO\textsubscript{2} abatement (Nordhaus 1994, Kolstad 1996, Pizer 1997), none rigorously explains this flatness in terms of the general properties of stock pollutants or clearly examines the link between this flatness and issues of policy choice. Presumably there are special features of stock pollutants that give rise to this phenomenon. This paper uncovers those features and evaluates how key variables influence the expected net benefits of alternative policies for controlling stock externalities.

choice of policies when benefits are a function of an accumulated stock, rather than the flow in a single period. Our results add several important new dimensions to the existing literature, with potential relevance to a wide variety of problems where policy interventions might be justified based on the existence of positive or negative stock externalities.

Stock pollutants such as carbon dioxide, other greenhouse gases, hazardous waste, pesticides in groundwater, and ozone depleting substances represent an important class of such problems. They are produced continually as a byproduct of economic activity, but—unlike pollutants such as volatile organic compounds—their harmful consequences are a function of a much larger stock accumulating in the environment, rather than an annual flow. The common element among these pollutants is that they persist in the environment for a long period of time. It is the atmospheric stock of greenhouse gases, for example, that is associated with global climate change, and thus the ultimate target of policies for controlling emissions of greenhouse gases. Other environmental and natural resource examples include policies addressing the destruction of wildlife habitat and species preservation; potential non-environmental applications include research and development, educational attainment, and monetary policy.

Regulating such stocks involves considerable uncertainty. The magnitude of their associated externalities and costs of control are often known only approximately and the regulation of contributions to a stock in any single year can affect the stock far into the future, which is itself uncertain. Further, even when uncertainties are resolved and firms gain a better idea of their costs, the regulator may not have access to this information and may not be able to elicit the truth from firms unless it is in the firms’ interest. This asymmetry of information between the regulator and firms producing the externality, in conjunction with a limited set of simple policy instruments assumed to be available to the regulator, are key features of the structure of the problem we address. Another key requirement is that policy is set before

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4 Most economic research on stock pollutants has focused on optimal control in a deterministic setting (Keeler, Spence, and Zeckhauser 1971; Plourde 1972; Smith 1972; Kitabatake 1989; Falk and Mendlesohn 1993). This removes the distinction among alternative instruments and sidesteps the issue of choosing an efficient instrument. Plourde and Yeung (1989) extend these models to incorporate stochastic elements, but they only consider uncertainty in the amount of pollution generated and the stock decay rate, that is, on the benefit side. Without cost uncertainty, price and quantity controls remain equivalent.

5 Laffont (1977) provides a careful description of the information structure assumed in policy choice problems formulated in the Weitzman tradition. Laffont also drew attention to the dual counterpart of Weitzman’s results, pointing out that prices on the consumption side would dominate the use of quantities when these were preferred to (continued)
uncertainties are resolved, while regulated firms can make output decisions after they learn more about their costs. Our focus on relatively simple and long-lived price and quantity policies is justified by the infeasibility and costliness of complex policies that entail continual readjustment or large amounts of information.⁷

When uncertainty exists about costs, and policies must be fixed before the uncertainty is resolved, priced-based policies will lead to distinctly different outcomes than quantity-based policies. Pollution taxes, for example, operate by encouraging firms to reduce emissions as long as the marginal cost of reducing an additional unit of pollution is less than the level of the pollution tax. The tax mechanism will lead to a range of possible emission levels across different cost outcomes—but will fix marginal cost at the level of the tax. Conversely, a quantity-based policy such as a tradeable permit system will fix the total volume of emissions with the equilibrium permit price determined by the marginal cost of the last reduction necessary to meet the emission constraint. The permit mechanism will therefore lead to a range of possible permit prices (and marginal costs) across different cost outcomes—but will lead to a fixed level of aggregate emissions. Different expected net benefits will therefore be associated with these alternate policies.

Weitzman’s remarkable insight was that, on economic efficiency grounds, a flat expected marginal benefit function (relative to marginal costs) favors prices, while a steep benefit function favors quantities.⁸ Intuitively, relatively flat marginal benefits imply a constant benefit per unit of control, suggesting that a tax could perfectly correct the externality. In contrast, steep prices on the production side. In the presence of externalities, however, there is generally not any available means for constructing such consumption side prices.

⁶ Iterative policy processes with truth-revealing procedures could, in principle, be designed to yield first-best outcomes even in the face of uncertainty and information asymmetries (Kwerel 1977; Dasgupta and Stiglitz 1977; Dasgupta, Hammond, and Maskin 1980).

⁷ It is possible, however, to combine price and quantity mechanisms to form superior hybrid policies that are relatively simple (Roberts and Spence 1976, Weitzman 1978), although these are rarely seen in practice. Nonetheless, the potential gains associated with using simple hybrid policies relative to pure price or quantity policies for controlling stock externalities warrants further research, which we have left for another paper due to space constraints. In addition, our focus on policies which are never adjusted may be overly restrictive for stock externality problems with very long time horizons. While it is obvious that additional flexibility in regulation over time can yield more efficient policies (e.g., by making regulations a function of the stock), it is less clear whether this flexibility will alter the relative performance of prices versus quantities.

⁸ An important footnote in Weitzman’s article is that correlation between uncertain costs and benefits can change this story. In particular, positive correlation tends to favor quantities and negative correlation tends to favor prices. Stavins (1996) shows that with plausible parameter values, this correlation may completely dominate Weitzman’s main point about relative slopes. In the case of stock externalities, we find that positive correlation over time in the costs of pollution control can have a similar effect on the basic Weitzman result.
marginal benefits imply a dangerous threshold that should be avoided at all costs—a threshold that is efficiently enforced by a quantity control. Despite its substantial insight, however, the Weitzman result remains a static story. Additional modeling is necessary as we turn to the specific question of controlling stock externalities, where consequences occur in a dynamic setting.

Using a simple analytical model that incorporates costs and benefits, stock decay, time discounting, and uncertainty, we uncover five important principles governing the choice of price-based policies relative to quantity-based policies for controlling stock externalities. First, the slower a stock decays (or the longer the stock persists), the more likely it is that quantity controls are preferred to price controls because deviations in flows under a price policy persist over long periods of time. Second, low discount rates give greater weight to these future deviations and thus reinforce the policy divergence over time, thereby favoring quantity controls.

Third, in a multi-period analysis results hinge on correlation across periods, just as Weitzman’s original results depend on the correlations of cost and benefit uncertainty at a single point time (Weitzman 1974, Stavins 1996). In particular positive correlation among cost shocks across time tend to decrease the net benefits of using prices, thereby favoring quantities. Fourth, controlling for the aforementioned effects, the relative slopes of the marginal benefits and costs of controlling the externality continue to be critical determinants of the efficiency of prices relative to quantities, with flatter marginal benefits and steeper marginal costs favoring prices. Fifth and finally, we find that the assumption of a globally quadratic benefit function almost guarantees that price controls will be more efficient than quantity based policies. In the case of climate change, these results suggest that the current focus on quantity-based policies, to the exclusion of policies that include price-like elements, may be inappropriate.

2. A Model of Stock Externality Regulation Under Uncertainty

In this section we develop an analytical model to explore alternative price and quantity mechanisms for regulating a stock externality, which could be either positive (ecosystems, biodiversity, education) or negative (toxic waste, greenhouse gases, pollution). For expositional clarity, we focus initially on the steady-state where under the optimal quantity regulation the
level of the stock is constant in all periods. Later we show that the results continue to hold in situations away from the steady state.

As a starting point for our modeling efforts, we use Weitzman’s original framework. This allows us to compare and contrast our results for stock externalities with his well-known results that are applicable to a single-period flow externality. To examine stock externalities, we make several modifications to his original framework, including omission of certain complicating features that turn out to be irrelevant for the final results. First, in our model benefits are a function of the stock and costs are a function of the flow, whereas in Weitzman’s model both costs and benefits were a function of the flow. Next, because changes in the stock level persist across time, it is necessary to set the model in a multi-period context. This dynamic context has several key features: time discounting, stock depreciation, and cost correlation. With benefits and costs occurring at different points in time, intertemporal prices, or discount factors, are required. In addition, the possibility of stock decay introduces a depreciation rate.

Finally, just as the results in the static analysis can depend on the correlation of benefit and cost uncertainty within the single period (Weitzman 1974, Stavins 1996), in a multi-period setting results will also hinge on correlation among costs in different periods. The idea of cost correlation deserves a brief comment since it turns out to have potentially enormous effects. One way to view cost uncertainty is as uncertainty about future technologies. In terms of pollution stocks, will we discover better abatement technologies or cleaner production processes, and how much cheaper would they be? Framed this way, it seems plausible that costs would exhibit a high positive correlation over time. In some future scenarios, we never discover these

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9 Our emphasis on the steady-state deserves a brief comment. Part of the appeal of the steady-state is its analytic simplicity; this facilitates obtaining closed form solutions for the relative advantage metric noted above. It also allows the development of considerable intuition regarding the nature of stock externalities that holds even away from the steady-state. In addition, for many stocks the existing level or “status quo” is often viewed as a reasonable balance of costs and benefits. Consider, for example, the focus on stabilizing greenhouse gas concentrations at recent levels and the interest in preserving existing ecosystems. For both these reasons, we believe it is useful to examine policies to regulate stock externalities at a steady-state.

10 First, we ignore uncertainty in the base level of costs and benefits (i.e., the constant terms in the cost and benefit functions) because Weitzman shows that it has no effect on the relative advantage of alternative policies. We also ignore benefit uncertainty because it does not affect the behavior of firms or individuals in response to a policy and therefore cannot affect the relative advantage of alternative policies unless it is correlated with costs. Weitzman (1974) and Stavins (1996) have explored the potential importance of such benefit-cost correlation in detail and it is not the focus of this paper; we therefore ignore benefit uncertainty in order to keep our model as simple as possible. We suspect that including correlation in costs and benefits in our model would have implications similar to those found in Weitzman’s original analysis, namely that positive correlation in benefits and costs will tend to favor quantity controls; the exact form of the consequences remains an issue for further research.
technologies and costs are consistently high. In others, unforeseen advances may permanently lower costs.\textsuperscript{11} An opposing argument could, however, be made for \textit{negative} correlation. In particular, it is not unreasonable to think that high costs, in fact, induce exactly the kind of innovation necessary to lower them (Newell, Jaffe, and Stavins 1998). Low costs, on the other hand, may introduce complacency which eventually leads to higher than expected costs. Nonetheless, we tend to think that forces leading to positive correlation are much more likely to dominate. In any event, cost correlation could be quite large.

Returning to the general setup of our model, for the basic metric for our policy comparisons we build on Weitzman’s method of computing the “comparative advantage” of price instruments over quantity instruments. This approach involves deriving the expected net benefits of regulation when firms or individuals are faced with particular policies (e.g., price or quantity controls) and then comparing these expected net benefits to those of a reference policy. In particular, we compute the expected net benefits of price and quantity policies and then calculate their difference $\Delta$:

$$\Delta = \mathbb{E}[NB_{\text{price}}] - \mathbb{E}[NB_{\text{quantity}}]$$

where $\mathbb{E}[\cdot]$ is the expectations operator. Thus, $\Delta > 0$ indicates that the optimal price-based policy performs better than the optimal quantity-based policy, while $\Delta < 0$ indicates the reverse. For what can be thought of as a single-period flow externality, Weitzman computed the comparative advantage of price policies relative to a quantity policies as

$$\Delta = \frac{\sigma^2}{2C''^2} (C'' - B''),$$

where $C''$ and $B''$ are, respectively, the absolute values of the slopes of the marginal cost and benefit functions and $\sigma^2$ is variance in costs. Thus, Weitzman found that price-based policies are preferred ($\Delta > 0$) so long as marginal benefits are relatively flat ($B'' < C''$).

\textsuperscript{11} Another way to think about the possibility of positive cost correlation is through uncertainty about the underlying level of uncontrolled emissions. Because this baseline flow is typically taken as given, uncertainty must instead be wrapped up in the cost shocks in the marginal cost schedule. If baseline emissions are closely related to the level of economic activity, as in the case of carbon emissions, high baseline emissions in one period suggest high levels in subsequent periods (e.g., high versus low growth scenarios), leading to considerable positive correlation in the cost shocks.
2.1 The Model

Following Weitzman's lead, we model costs and benefits as quadratic relations written in terms of deviations from the optimal quantity:

\[ B(S_t) = b_0 + b_1 (S_t - \hat{S}) - \frac{1}{2} b_2 (S_t - \hat{S})^2 \]

(3)

\[ C(q_t, \theta_t) = c_0 + (c_1 + \theta_t) (q_t - \hat{q}) + \frac{1}{2} c_2 (q_t - \hat{q})^2 , \]

(4)

where \( t \) indexes time, \( S_t \) is the stock level, \( q_t \) is the quantity of policy action (i.e., contribution to the flow), \( \hat{S} \) and \( \hat{q} \) are the optimal steady-state values of those variables, \( b_n \) and \( c_n \) are parameters of the functions, and \( \theta_t \) is a shock to the cost function with zero mean, variance \( \sigma^2 \), and correlation \( \rho \) across time. Augmentation of the stock is achieved through positive values for \( q \) and reductions through negative values for \( q \). Both \( b_2 \) and \( c_2 \) will be positive assuming that benefits are concave and costs are convex. Positive values for \( b_1 \) and \( c_1 \) represent cases of a positive externality, indicating that higher values of the stock yield positive marginal benefits, but require increasing marginal costs. Likewise, negative values for \( b_1 \) and \( c_1 \) represent cases of a negative externality.

The dynamic nature of the stock is represented by the following accumulation equation:

\[ S_t = (1 - \delta) S_{t-1} + \bar{q} + q_t , \]

(5)

where \( \delta \) is the depreciation or decay rate of the stock and \( \bar{q} \) is an exogenous flow into the stock each period in the absence of any policy, and can be either positive or negative. The depreciation rate can take on values \( 0 \leq \delta \leq 1 \) representing cases ranging from a “pure stock externality” that persists forever \( (\delta = 0) \) to a “flow externality” \( (\delta = 1) \) that replicates the traditionally analyzed case. In order to accommodate benefits and costs that occur at different points in time, we also introduce a discount rate \( r \). The expression for the net benefit \( NB \)

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12 This is pragmatic because it simplifies many of the analytic expressions. It also suggests that one can view these strong functional form assumptions as approximations, an issue about which we remain agnostic (Malcomson 1978, Weitzman 1978).

13 The assumption that benefits are concave guarantees that the marginal benefits diminish with increased reductions of a bad stock and increased contributions to a good stock.

14 One could imagine the case of a biological population where \( \delta \) would be negative.

15 From Equation (5), it must be the case that \( \delta \bar{S} = \bar{q} + \hat{q} \) in order for \( \hat{S} \) and \( \hat{q} \) to be a feasible steady-state solution.
associated with paths for the chosen policy \( \{q_t\} \) and realized cost shocks \( \{\theta_t\} \) can now be written as

\[
NB = \sum_{t=1}^{\infty} \frac{B(S_t) - C(q_t, \theta_t)}{(1+r)^t} \quad \text{such that} \quad S_t = (1 - \delta)S_{t-1} + \bar{q} + q_t.
\] (6)

2.2 Expected Net Benefits of the Optimal Quantity-Based Policy

To determine the optimal steady state quantity policy and its associated net benefits, we begin by maximizing the expected discounted net benefits of stock control with respect to the policy path \( \{q_t\} \), subject to the stock accumulation relationship given in Equation (5). In the Appendix we show that this leads to a very simple expression for the expected net benefits associated with the optimal quantity policy, namely

\[
E[NB_{\text{quantity}}] = \frac{1}{r} (b_0 - c_0).
\] (7)

Equation (7) represents the net present value of an infinite stream of \( b_0 - c_0 \) expected net benefits each period.

2.3 Expected Net Benefits of the Optimal Price-Based Policy

Since costs and benefits depend on \( q_t \) and \( S_t \), the first step in calculating the optimal price policy is to determine the flow quantity that would result from a particular price policy. This response function \( q(P_t, \theta_t) \) depends on both the price \( P_t \) set by the regulator and the cost shock \( \theta_t \). Assuming firms respond to price policy \( P_t \) by equating it to the marginal cost of output, \( c_1 + \theta + c_2(q_t - \hat{q}) \), the response function will be given by

\[
q(P_t, \theta_t) = \hat{q} + \frac{P_t - c_1 - \theta_t}{c_2}.
\] (8)

Noting that when \( P_t = c_1 \), \( E[q_t] = \hat{q} \) and therefore \( E[S_t] = \hat{S} \), in the Appendix we demonstrate that this is indeed the optimal steady-state price policy. Intuitively, this price policy simply continues to satisfy the first order conditions satisfied by the optimal quantity policy since marginal benefits and costs are linear. Based on \( P_t = c_1 \), the price policy will take the form of a subsidy on \( q \) in the case of a positive externality (because \( c_1 > 0 \)) and a tax on \( q \) in the case of a negative externality (because \( c_1 < 0 \)).
Calculating the expected net benefits associated with the optimal price policy is complicated by fluctuations in $q_t$ and $S_t$ that occur under price regulation. In the Appendix, we show that

$$E[NB_{\text{price}}] = \frac{1}{r} \left( b_0 - \frac{b_2 \sigma^2}{2c_2^2} \left( \frac{1 + r}{\delta(2 - \delta) + r} \right) \left( \frac{1 + r + \rho(1 - \delta)}{1 + r - \rho(1 - \delta)} \right) - c_0 + \frac{\sigma^2}{2c_2} \right).$$  \hspace{1cm} (9)

where we now have $\sigma^2$ and $\rho$ reflecting the variation and correlation (across time) of cost shocks, respectively. We provide a full interpretation of the terms in this expression through the special cases discussed in the next section and in greater detail in the Appendix.

2.4 The Relative Advantage of Price-Based and Quantity-Based Policies

Based on Equations (7) and (9), we compute the relative advantage of price controls over quantity controls as

$$\Delta = \frac{c_2 - b_2}{r} \left( \frac{1 + r}{\delta(2 - \delta) + r} \right) \left( \frac{1 + r + \rho(1 - \delta)}{1 + r - \rho(1 - \delta)} \right).$$  \hspace{1cm} (10)

To develop intuition for this expression, we consider several special cases while comparing Equation (10) to Weitzman's original expression $\Delta = (\sigma^2 / 2c_2^2)(c_2 - b_2)$. We then offer several general results.

**Flow Externality.** First consider the case of a flow externality, where $\delta = 1$. Equation (10) reduces to Weitzman’s original expression, except for the present value factor $1/r$ multiplying the entire expression. This factor captures the fact that we are considering an infinite horizon of time periods identical to the single period case covered by Weitzman.

**Pure Stock Externality and No Cost Correlation.** If $\delta = 0$ and $\rho = 0$, corresponding to a pure stock externality with no correlation of cost shocks, $b_2$ is multiplied by an additional present value factor $(1 + r)/r$. This factor enters because the costs of changing the flow in a given period occur only in that period while the associated benefits persist into the indefinite future. It indicates that the slope of the marginal cost function $c_2$ must be compared with the slope of the marginal benefit function summed over that infinite horizon $b_2(1 + r)/r$, rather than just $b_2$ as in the case of flow externality. Depending on the discount rate, this “horizon effect” has the potential to greatly increase the relative importance of marginal benefits. In particular, regardless of how $b_2$ and $c_2$ compare, if we care enough about the future (e.g., $r$ near zero) it is
always possible that $b_2(1+r)/r > c_2$, implying a preference for quantity controls. In the climate debate, for example, this is one possible explanation for the persistent emphasis on quantity controls.

**Pure Stock Externality and Perfect Cost Correlation.** The potential importance of cost correlation to policy choice is highlighted by the case where $\delta = 0$ and $\rho = 1$, corresponding to a pure stock externality with perfect positive correlation of cost shocks. In this case there is another factor $(2+r)/r$ multiplying $b_2$ which arises because deviations in one period not only persist into the indefinite future, but they are indicative of future deviations which compound the initial one. This “cost correlation effect” has the potential to increase the effective slope of the marginal benefit function above and beyond the horizon effect. In fact, it can introduce a factor of up to 40 to 200 based on discount rates ranging from 5 percent down to 1 percent.

**General Results.** Broadening beyond these special cases, we can make several general observations based on Equation (10); we leave the derivations for the Appendix.

1. **More steeply sloped marginal costs favor price controls, while more steeply sloped marginal benefits favor quantity controls.** This observation reaffirms that Weitzman’s original result—that the relative slopes of marginal costs and benefits are fundamental to policy choice—continues to hold in the dynamic context of a stock externality. Quantities are preferred in cases where strong nonlinearities or thresholds exist, leading to steep marginal benefits; less curvature in benefits, on the other hand, tends to favor prices.

2. **Lower depreciation rates for the stock favor quantity controls.** Intuitively, less depreciation allows the variability in the flow rate introduced by a price policy to persist longer. Greater variability lowers expected benefits because benefits are concave.

3. **Lower discount rates favor quantity controls.** Because benefits associated with activities in a given period persist while costs are contained in each period, a lower discount rate increases the importance of those future benefits. The factors multiplying $b_2$ rise, thereby lowering $\Lambda$ and indicating that quantity controls are more attractive.

4. **Positive correlation in costs across time favors quantity controls.** When costs are positively correlated across time, deviations in the stock arising under a price mechanism accumulate rather than canceling out. This exacerbates variation in the stock level and lowers benefits, as was the case with lower depreciation rates.
3. Further Results

3.1 Policy Choice Away from the Steady State

The discussion so far has focused on choosing among policies when the expected marginal benefits and marginal costs of keeping the stock fixed at the optimal level roughly balance. In this section we consider the case where it is not desirable to keep the stock constant and the optimal policy involves either an increase or decrease in the stock. Such periods of transition are relevant during the early stages of policy action when regulation moves the stock toward the level desired in the long run. Perhaps surprisingly, all of our results from the previous section continue to hold so long as we maintain the assumption of quadratic costs and benefits away from the initial stock and flow levels. As shown in the Appendix, the expression representing the comparative advantage of price over quantity controls is unchanged regardless of the initial stock level.

Intuitively, optimal price controls always match the quantity control in expected value terms. In turn, the expected marginal costs and benefits are also equal under the two types of policy because the marginal expressions are linear in the stock and flow variables. A solution to the quantity optimization also solves the price optimization. The welfare difference between the policies arises in the expected squared quantity deviations that occur with a price policy. These deviations reduce costs because firms do less when it is expensive and more when it is cheaper, but on average they do exactly the same. Nonetheless, the deviations decrease benefits according to Jensen's inequality because benefits are concave. The decrease in costs and benefits under the price policy depends only on the deviations of flow level from its expected value. In this way, the difference between the policies is independent of the initial stock level.

It is important to recognize, however, that once we examine situations outside the steady state, we lean more heavily on our functional form assumptions—functional forms which must now hold over a much wider range of values. At the same time, assuming globally quadratic costs and benefits allows us to make additional observations about the likely efficiency of price versus quantity controls.

3.2 An Argument Favoring Price-Based Policies for Stock Externalities

The first question we ask is how the parameters of the cost and benefit functions are restricted in the case of globally quadratic forms. The crux is that we must require marginal benefits and costs to be positive for either reductions of a bad stock, or increases in a good stock,
at all plausible levels of the chosen flow \( q \) and resulting stock \( S \). For both clarity and practicality, we focus on the case of bad stock although similar results can be derived in the case of good stock.\(^{16}\)

Writing our global benefit model as

\[
B(S_t) = b_0 + b_1 S_t - \frac{1}{2} b_2 S_t^2
\]

in addition to our original concavity restriction \((b_2 > 0)\), the implication for a bad stock is that \( b_1 \leq 0 \); that is, there are marginal damages associated with any positive level of the stock.\(^{17}\) In our earlier treatment of the steady state we made a similar restriction about marginal effects, but only at the optimal steady-state level. We now make the requirement for all positive values of the stock.

In our global model of costs

\[
C(q_t) = c_0 + (c_1 + \theta_t) q_t + \frac{1}{2} c_2 q_t^2
\]

beyond the convexity restriction \((c_2 > 0)\), we require \( c_1 = 0 \) so that the expected marginal cost is zero at zero reductions. We require that this be exactly zero rather than non-positive because we presume that the exogenous level of emissions \( \bar{q} \) is exactly the total flow level associated with zero marginal costs. Otherwise, there would be an incentive to change the flow level even in the absence of regulation.

Applying these restrictions, a surprising result is that in the absence of correlation of cost shocks, price controls are preferred to quantity controls so long as the optimal stock is at least 50 percent larger than the annual flow. The intuition for this result can be seen in Figure 1, which shows the steady-state equilibrium between the marginal costs and benefits of current period controls. Noting that the horizontal axis measures negative quantities, marginal costs rise from zero to the equilibrium price over a distance of \( \hat{q} \). At that level of net flow, namely \( \bar{q} - |\hat{q}| \) (\( \hat{q} \) is negative), the stock is constant at the steady state level \( \hat{S} \). Therefore reducing the flow by an additional \( \hat{S} \) will reduce the stock in the current period to zero. Even at this point, however,

\(^{16}\) Good and bad stocks refer to whether we are planning to expend resources to increase the stock or decrease the stock, respectively.

\(^{17}\) This might seem incongruous with our motivating example, greenhouse gases, which have a natural concentration in the atmosphere which we would presume to be beneficial. In this case, one can view \( S \) as the stock above the “natural level” and \( \bar{q} \) as the exogenous annual flow above the amount that can be assimilated at the natural stock level.
the marginal benefits of further reductions will still be positive because (i) marginal benefits in the current period are non-negative by our restriction on \( b_1 \); and (ii) marginal benefits in future periods (associated with reductions in the current period) are positive since the stock in future periods will still be positive.\(^{18}\)

Since the horizontal distance over which marginal costs rise to the equilibrium price, namely \( \hat{q} \), is smaller than the horizontal distance over which marginal benefits rise to the equilibrium price, namely \( \hat{S} \), the slope of the marginal cost curve is necessarily steeper than the marginal benefit curve. The marginal benefit curve has a slope of \( \frac{1+r}{r+\delta} b_2 \) reflecting the discounted and depreciated sum of marginal benefits in all future periods arising from reductions in the current period (see Appendix). The marginal cost curve has a slope of \( c_2 \) reflecting costs of reductions in the current period. By showing that \( c_2 > \frac{1+r}{r+\delta} b_2 \) we have almost met the condition, in the absence of cost correlation across time, for price controls to be preferred to quantity controls given by Equation (10). By requiring that \( \hat{S} \) be 50 percent larger than \( \hat{q} \), we

---

\(^{18}\) Because the diagram represents the steady state, by assumption the net flow in all future periods is \( \bar{q}+\hat{q} \).
can show that \( c_2 > b_2 \frac{1+r}{r+2\delta - \delta^2} \), which is exactly the condition required for prices to dominate quantities for controlling a stock externality. The strength of these arguments in favor of price controls is arguably mitigated by our assumption of globally quadratic damages and, perhaps more importantly, the absence of correlation in cost shocks. As noted earlier, cost correlation can easily overwhelm arguments based on the relative slopes of marginal costs and benefits.

Other authors have conjectured about this type of result based on recent analyses involving global climate change. Nordhaus (1994) observes (in a footnote) that damages in his climate change policy simulations are essentially a linear function of emissions and, based on a Weitzman-style argument, that this implies a preference for price controls. He then suggests that this preference might extend to stock pollutants more generally. Pizer (1997) directly investigates the price versus quantity question for climate change using monte carlo simulations. Like Nordhaus and Kolstad (1996), he observes linear damages, but is further able to demonstrate that price policies indeed lead to much better welfare outcomes. Finally, McKibbin and Wilcoxen (1997) argue that the absence of an obvious benefit to stabilizing either the flow or stock weighs heavily in favor of a price mechanism. None of these authors, however, are able to explain the conditions under which their observed results will continue to hold.

4. Conclusion

Seminal work by Weitzman made an important distinction between otherwise equivalent price and quantity controls. Price mechanisms are more efficient when marginal benefits are relatively flat and quantity mechanisms are more efficient when benefits are relatively steep. This intuition carries over, with modification, to the case of stock externalities considered in this paper. Flatter benefits continue to favor price controls, but Weitzman's intuition is complicated in several ways. It is no longer a simple relative slopes argument. The slope of the cost curve must instead be compared to an adjusted measure of marginal benefit. This adjusted measure takes into account discounting, depreciation, and correlation of cost shocks.

In particular, lower discount rates, lower depreciation rates, and higher correlation among cost shocks all favor quantity controls. Because the benefits of control in one period carry over into future periods, lower depreciation and discount rates raise the importance of benefits, as well as the adjusted measure of marginal benefit. In addition, greater cost correlation increases the variability of the stock level under price controls and, due to the concavity of benefits, leads to a
greater preference for quantity controls. Finally, we show that under the assumption of globally quadratic benefits, price controls are likely to be preferred unless the optimal stock level is roughly the same as the annual uncontrolled flow. Quadratic benefits and large stock/flow ratios imply extremely flat marginal benefits.

These results are potentially applicable to a wide range of market failures involving stock externalities. In addition to the obvious application to stock pollutants, one can view species preservation, land use policy, education, monetary policy, and research as areas where policymakers wish to regulate a stock-like externality. As either a normative policy guide or positive explanation of behavior, this analysis provides a useful framework for comparing alternative policy instruments. Our initial motivation for this work, for instance, was the problem of controlling greenhouse gas emissions in response to the threat of global warming. Given that the stock of greenhouse gases in the atmosphere is an order of magnitude greater than the annual global flow, in the absence of evidence of highly non-linear damages or strong positive correlation and very low discount rates, these results argue strongly in favor of price-based controls. This is consistent with the observations of Nordhaus(1994), McKibbin and Wilcoxen (1997), Pizer (1997), and others. Our conclusions suggest that the current focus on quantity-based policies, to the exclusion of policies that include price-like elements, may be inappropriate.
Appendix

A.1 Derivation of the Optimal Quantity Policy

The objective function for the regulator is expected net benefits given by Equation (6). We derive the optimal quantity policy by maximizing expected net benefits with respect to the policy path \( \{q_t\} \) subject to the stock accumulation relationship given by Equation (5). The Lagrangian form of this optimization problem is

\[
\max E \left[ \sum_{t=1}^{\infty} \frac{B(S_t) - C(q_t, \theta_t) - \lambda_t (S_t - S_{t-1} (1-\delta) - \bar{q} - q_t)}{(1+r)^t} \right],
\]

which in turn generates two first order conditions in addition to the stock constraint:

\[
-E[C'(q_t, \theta_t)] + \lambda_t = 0 \quad \text{and} \quad E[B'(S_t)] - \lambda_t + \frac{1-\delta}{1+r}\lambda_{t+1} = 0.
\]

The first condition reduces to \( c_1 + c_2 (q_t - \bar{q}) = \lambda_t \) and the second to \( b_1 + b_2 (S_t - \hat{S}) = \lambda_t - \frac{1-\delta}{1+r}\lambda_{t+1} \). By construction, \( \hat{S} \) and \( \hat{q} \) are the expected optimal steady state values, and \( \lambda_t = \lambda_{t+1} \) in the steady state. The two first order conditions can therefore be solved to yield

\[
E[C'(q_t, \theta_t)] = \left( \frac{1+r}{r+\delta} \right) E[B'(S_t)]
\]

or

\[
c_1 = \left( \frac{1+r}{r+\delta} \right) b_1.
\]

At the optimum, the expected increase in costs associated with a marginal increase in activity in any period \( E[C'(q_t, \theta_t)] \) must equal the sum of the discounted, depreciated expected marginal benefits,

\[
E[B'(S_t)] + \frac{1-\delta}{1+r} E[B'(S_{t+1})] + \left( \frac{1-\delta}{1+r} \right)^2 E[B'(S_{t+2})] + \cdots.
\]

That is, a marginal increase in expected costs in one period must be balanced against the marginal benefits accruing in both current and future periods. The benefits in future periods depreciate because any change in the stock depreciates along with the stock, and become discounted because we assume future income is valued less than current income.
One way to view the result in (13) is to recognize that for every quantity \( q_t \), there is an equilibrium stock level given by \( \delta S_t = \bar{q} + q_t \). This equilibrium activity level/stock combination will be optimal only if the expected slope of the cost function at \( q_t \) equals the magnitude of the slope of the benefit function at \( S_t \). Calling those values \( \hat{q} \) and \( \hat{S} \), respectively, and writing benefits and stocks according to Equations (3) and (4) we have exactly condition (13) satisfied at that point.

Given the optimal quantity policy involves \( q_t = \hat{q} \) and \( S_t = \hat{S} \) every period, expected net benefits according to Equation (6) are given by

\[
E\left[ NB_{\text{quantity}} \right] = E\left[ \sum_{t=1}^{\infty} \frac{B(\hat{S}) - C(\hat{q}, \theta_t)}{(1 + r)^t} \right] = E\left[ \sum_{t=1}^{\infty} \frac{b_0 - c_0 - \theta_t}{(1 + r)^t} \right] = \frac{b_0 - c_0}{r}.
\]

### A.2 Derivation of the Optimal Price Policy

To compute the optimal price policy, we employ the Lagrangian given by Equation (12) with two changes: (i) the choice variable is now \( \{ P_t \} \), rather than \( \{ q_t \} \); and (ii) we substitute the response function \( q(P_t, \theta_t) = \hat{q} + \frac{P_t - c_1 - \theta_t}{c_2} \) for \( q_t \). The resulting first order conditions are

\[
E[\hat{q}'(P_t)] = 0 \quad \text{and} \quad E[B'(S_t)] = 1 - \frac{\delta}{1 + r} \lambda_{t+1} = 0.
\]

Because \( q'(P_t) = 1/c_2 \) is not a random variable, the expectation operator passes through as before. The firm equalizes price and marginal cost, yielding \( P_t = \lambda_t \) from the first condition and \( b_1 + b_2 (E[S_t] - \hat{S}) = \lambda_t - \frac{1 - \delta}{1 + r} \lambda_{t+1} \) from the second. These conditions will be satisfied if \( P_t = c_1 \), yielding \( E[q_t] = \hat{q} \), \( E[S_t] = \hat{S} \), and \( c_1 = (\frac{\lambda_t}{\lambda_{t+1}})b_1 \).

To compute expected net benefits, we begin by defining the quantity deviations \( dq_t = q_t - \hat{q} \) and deviations of the stock levels \( dS_t = S_t - \hat{S} \) from the optimal quantity levels. It is assumed that at \( t = 0 \), the system is in a steady state with \( S_0 = \hat{S} \) and \( dS_0 = 0 \). The response function \( q_t(P_t, \theta_t) \) yields an expression for the quantity deviation given the optimal price policy \( P_t = c_1 \):
\[ dq_t = \frac{P_t - c_1 - \theta_t}{c_2} = -\frac{\theta_t}{c_2}. \]

This allows us to compute expected costs in each period:

\[
E[C(q_t, \theta_t)] = E[c_0 + (c_1 + \theta_t)dq_t + \frac{1}{2} c_2 dq_t^2] = E\left[c_0 - (c_1 + \theta_t) \frac{\theta_t}{c_2} + \frac{1}{2} c_2 \left(-\frac{\theta_t}{c_2}\right)^2\right] = c_0 - \frac{\sigma^2}{2c_2}.
\]

The discounted value of all costs from period one forward is therefore equal to

\[
\frac{1}{r} \left(c_0 - \frac{\sigma^2}{2c_2}\right).
\]

Computing expected benefits is more complex. Beginning in the first period, deviations in the activity level from its steady-state value create deviations in the stock from its steady state value. These deviations persist into future periods when depreciation is less than 100 percent. In particular, the stock in period \( t \) will be equal to

\[
S_t = \hat{S} + (1 - \delta)^{t-1} dq_1 + (1 - \delta)^{t-2} dq_2 + \ldots + dq_t,
\]

\[
= \hat{S} - (1 - \delta)^{t-1} \frac{\theta_1}{c_2} - (1 - \delta)^{t-2} \frac{\theta_2}{c_2} - \ldots - \frac{\theta_t}{c_2}.
\]

The benefits in any period \( t \) are then given by

\[
E[B_t] = E\left[b_0 + b_1 dS_t - \frac{1}{2} b_2 dS_t^2\right] = b_0 - \frac{1}{2} b_2 E\left[dS_t^2\right]
\]

\[
= b_0 - \frac{1}{2} b_2 E\left[\left(- (1 - \delta)^{t-1} \frac{\theta_1}{c_2} - (1 - \delta)^{t-2} \frac{\theta_2}{c_2} - \ldots - \frac{\theta_t}{c_2}\right)^2\right].
\]

\[
= b_0 - \frac{1}{2} b_2 U_i(t)
\]

where we the expression \( U_i(s) \) represents the expectation of \( dS_t^2 \) assuming that shocks prior to period \( t - s + 1 \) equal zero. Examining this expression,
\[ U_t(s) = \mathbb{E}\left[ - (1 - \delta)^{s-1} \frac{\theta_{t-s+1}}{c_2} - (1 - \delta)^{s-2} \frac{\theta_{t-s+2}}{c_2} - \cdots - \frac{\theta_t}{c_2} \right]^2 \]

\[ = \frac{1}{c_2^2} \mathbb{E}\left[ \sum_{r=0}^{s-1} (1 - \delta)^{2r} \theta_r^2 + \sum_{r=1}^{s-1} \sum_{q=0}^{r-1} (1 - \delta)^{r+q} \theta_r \theta_{t-q} \right] \]

\[ = \frac{1}{c_2^2} \left( \sum_{r=0}^{s-1} (1 - \delta)^{2r} \sigma^2 + \sum_{r=1}^{s-1} \sum_{q=0}^{r-1} (1 - \delta)^{r+q} \rho^{r-q} \sigma^2 \right) \]

\[ = U(s) \]

where the last line reflects the fact that the expectation only depends on how many periods we go back, not what period we start going back from. Before simplifying the expression for \( U(s) \), we write out the expression for the entire sequence of benefits:

\[ \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \mathbb{E}[B_t] = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \mathbb{E}[B_t] \]

\[ = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (b_0 - \frac{1}{2} b_2 U(t)) \]

\[ = \frac{b_0}{r} - \frac{1}{2} b_2 \sum_{t=1}^{\infty} \frac{U(t)}{(1+r)^t} \]

and instead look at simplifications of the expression \( \sum_{t=1}^{\infty} \frac{U(t)}{(1+r)^t} \). Namely,

\[ \sum_{t=1}^{\infty} \frac{U(t)}{(1+r)^t} = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \] \( U(t) - U(t-1) + U(t-1) - U(t-2) + \cdots + U(1) - U(0) \)

\[ = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (U(t) - U(t-1)) + \sum_{t=2}^{\infty} \frac{1}{(1+r)^t} (U(t-1) - U(t-2) + \cdots + U(1) - U(0)) \]

\[ = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (U(t) - U(t-1)) + \frac{1}{(1+r)} \sum_{t=2}^{\infty} \frac{1}{(1+r)^t} (U(t-1) - U(t-2) + \cdots + U(1) - U(0)) \]

\[ = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (U(t) - U(t-1)) + \frac{1}{(1+r)} \sum_{t=1}^{\infty} \frac{U(t)}{(1+r)^t} \]

\[ = \frac{1+r}{r} \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (U(t) - U(t-1)) \]

where we define \( U(0) = 0 \).
Intuitively, $U(s)$ reflects the expected squared stock deviation, assuming no deviations in the flow $s$ or more periods in the past. The expression $U(s + 1) - U(s)$ is then the additional contribution to the expected squared stock deviation allowing for a non-zero flow deviation $s$ periods in the past. With this intuition, the expression

$$\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (U(t) - U(t-1))$$

can be viewed as the (additional) contribution to stock deviations in all future periods arising from shocks in the first period, discounted appropriately. We now look at how this expression can be further simplified:

$$\sum_{t=1}^{\infty} \frac{1}{c_2^2} \sum_{r=1}^{\infty} \frac{1}{(1+r)^t} \left\{ \frac{\sum_{r=0}^{t-1} (1-\delta)^{\beta r} \sigma^2 + 2 \sum_{r=1}^{t-1} \sum_{q=0}^{t-1} (1-\delta)^{\beta r \rho^q} \sigma^2}{1- \sum_{r=0}^{t-1} (1-\delta)^{\beta r} \sigma^2 + 2 \sum_{r=1}^{t-2} \sum_{q=0}^{t-2} (1-\delta)^{\beta r \rho^q} \sigma^2} \right\}$$

$$= \frac{\sigma^2}{c_2^2} \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \left\{ \frac{(1-\delta)^{2\beta (t-1)} + 2 \sum_{q=0}^{t-2} (1-\delta)^{\beta (t-1+q)} \rho^{\beta -1+q}}{1- \sum_{r=0}^{t-1} (1-\delta)^{\beta r} \sigma^2 + 2 \sum_{r=1}^{t-2} \sum_{q=0}^{t-2} (1-\delta)^{\beta r \rho^q} \sigma^2} \right\}$$

The first sum in the last line is easily computed as

$$\sum_{t=0}^{\infty} \frac{(1-\delta)^{2\beta t}}{(1+r)^t} = \frac{1}{1-r} \frac{1+r}{1+r - (1-\delta)^{2\beta r}} = \frac{1+r}{r + 2\delta - \delta^2}.$$

The second sum is computed by noting that

$$\sum_{t=2}^{\infty} \sum_{q=0}^{t-1} (1-\delta)^{\beta (t-1+q)} \rho^{\beta -1+q} \sigma^2 = \frac{1}{(1+r)} \sum_{t=1}^{\infty} \sum_{q=0}^{t-1} (1-\delta)^{\beta (t-1)} \rho^{\beta -1+q} \sigma^2$$

$$= \frac{1}{(1+r)} \sum_{t=1}^{\infty} \frac{(1-\delta)^{2\beta (t-1)} + 2 \sum_{q=0}^{t-2} (1-\delta)^{\beta (t-1+q)} \rho^{\beta -1+q}}{(1+r)^t}$$

$$= \frac{\rho (1-\delta)}{(1+r)^2 (r + 2\delta - \delta^2)} \sum_{t=0}^{\infty} \frac{(1-\delta)^{2\beta t}}{(1+r)} + \frac{1}{(1+r) \sum_{t=2}^{\infty} \sum_{q=0}^{t-2} (1-\delta)^{\beta (t-1+q)} \rho^{\beta -1+q} \sigma^2}{(1+r)^t}$$

$$= \frac{\rho (1-\delta)}{(1+r)(r + 2\delta - \delta^2)} \frac{1+r - (1-\delta)\rho}{(1+r - \rho + \rho \delta)} = \frac{\rho (1-\delta)}{(1+r)(r + 2\delta - \delta^2)(1+r - \rho + \rho \delta)}$$
Yielding

\[
\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} (U(t) - U(t-1)) = \frac{\sigma^2}{c^2} \left( \frac{1}{r + 2\delta - \delta^2} + 2 \frac{\rho(1-\delta)}{(r + 2\delta - \delta^2)(1+r - \rho + \rho\delta)} \right)
\]

or

\[
\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \mathbb{E}[B_t] = \frac{1}{r} \left( b_0 - \frac{b_2\sigma^2}{2c^2} \left( \frac{1+r}{\delta(2-\delta) + r} \right) \left( \frac{1 + r + \rho(1-\delta)}{1+r - \rho(1-\delta)} \right) \right).
\]

Combining this with the expression for costs yields an expression for expected net benefits associated with the price policy:

\[
\mathbb{E}[NB_{\text{price}}] = \frac{1}{r} \left( b_0 - \frac{b_2\sigma^2}{2c^2} \left( \frac{1+r}{\delta(2-\delta) + r} \right) \left( \frac{1 + r + \rho(1-\delta)}{1+r - \rho(1-\delta)} \right) - c_0 + \frac{\sigma^2}{2c^2} \right).
\]

The last term represents the decrease in expected costs arising from the fact that the price control relaxes the quantity control when costs are high and increases it when costs are low—reducing average costs. On the other hand, the second term shows that this variation in \(q_t\) lowers expected benefits because benefits are a concave function. In particular, deviations in the stock lead to an expected dead weight loss equal to \(b_2\sigma^2/2c^2\) due to new deviations in each period. The dead weight loss generated in any particular period then depreciates in the future. Because the dead weight loss depends on the squared deviation, the factor \((1-\delta)^2\) appears in the expression alongside the discount rate \((1+r)\)—the expression

\[
\frac{b_2\sigma^2}{2c^2} \left( \frac{1}{\delta(2-\delta) + r} \right) \left( \frac{1 + r + \rho(1-\delta)}{1+r - \rho(1-\delta)} \right)
\]

represents the net present value of expected losses arising from quantity deviations in a particular period.

When positive correlation in the cost shocks exists \((\rho > 0)\), the expected loss of benefits rises. With positive correlation, deviations tend to build on each other rather than canceling each other out (when \(\rho < 0\)). The exact form of the correlation term arises from the fact that an activity shock of \(-\theta_t/c_2\) in any period \(t\) yields an expected benefit loss of \(2\rho(1-\delta)b_2\sigma^2/c_2^2\) in period \(t+1\), an expected benefit loss of \(2\left(\rho(1-\delta)^3 + \rho^2(1-\delta)^2\right)b_2\sigma^2/c_2^2\) in period \(t+2\), and, generally, an expected benefit loss of

\[
2\left(\rho(1-\delta)^{2i-1} + \rho^2(1-\delta)^{2i-2} + \cdots + \rho^i(1-\delta)^i\right)b_2\sigma^2/c_2^2
\]
in period $t + s$. Discounting and summing these expected benefit losses yields the correlation term in Equation (9). The $1/r$ factor in front represents the fact that, so far, we have been discussing the consequences of an activity deviation in a single period; allowing for such deviations in all periods yields the $1/r$ factor.

A.3 Derivation of the Effects of Discounting, Depreciation, and Cost Correlation

The effect of variables on the relative advantage of price versus quantity policies can be found by determining whether increases in the variables raise or lower the factors multiplying $b_2$ in $\Delta$, which we define as $\Omega$, where

$$
\Omega = \frac{1 + r}{\delta (2 - \delta) + r} \left( \frac{1 + r + \rho (1 - \delta)}{1 + r - \rho (1 - \delta)} \right). 
$$

To determine the effect of the discounting, stock depreciation, and cost correlation on $\Delta$, we take the derivative of $\Omega$ with respect to $r$, $\delta$, and $\rho$ and determine the sign of the resulting expression assuming that $0 < r < 1$, $0 < \delta < 1$, and $-1 < \rho < 1$.

**The Effect of Time Discounting.**

$$
\frac{d\Omega}{dr} = -\frac{(1 - \delta)((1 + r)^2 - \rho^2 (1 - \delta)^3)}{(1 + r + \rho (1 - \delta))^2 (1 + r - \rho (1 - \delta))}. 
$$

If $\rho$ is positive, although there appears to be some indeterminacy in the sign of this expression due to the term $-\rho^2 (1 - \delta)^3$, one can show that $(1 + r)^2 - \rho^2 (1 - \delta)^3 > 0$, implying that the expression is negative. Lower (higher) discount rates therefore increase (decrease) the weight on $b_2$, thereby increasing (decreasing) the advantage of quantity-based (price-based) policies as long as cost correlation is positive. If cost correlation is negative the effect is indeterminate.

**The Effect of Stock Depreciation.**

$$
\frac{d\Omega}{d\delta} = -\frac{2(1 + r)((1 + r)^2 - \rho^2 (1 - \delta)^3 + \rho (1 + r)(r + \delta (2 - \delta)))}{((r + \delta (2 - \delta))^2 (1 + r - \rho (1 - \delta))^2}. 
$$

If $\rho$ is positive, the potential indeterminacy of Equation (16) is the same as for Equation (15), implying that the expression is negative. Lower (higher) depreciation therefore favors quantities (prices) as long as cost correlation is positive; with negative cost correlation the effect is again indeterminate.
The Effect of Cost Correlation.

\[
\frac{d\Omega}{d\rho} = \frac{2(1 + r)^2 (1 - \delta)}{\left( r + \delta(2 - \delta)(1 + r - \rho(1 - \delta))^2 \right)}.
\]  

(17)

All terms are positive, indicating that greater positive (negative) cost correlation increases the advantage of quantity-based (price-based) policies.

A.4 Proof that the Relative Advantage $\Delta$ Remains the Same Away from the Steady State

We prove the result in three steps: (i) we assume a optimal quantity policy exists; (ii) we show that the corresponding price optimum leads to expected quantity and stock values that are the same as the optimal quantity policy; and (iii) we show that the expression $\Delta$ depends only on the cost shocks, the discount and depreciation rate, and the curvature of the cost and benefit functions, and not the initial level of the stock. Since it does not depend on the initial level of the stock, the $\Delta$ computed at the steady state must be the same as $\Delta$’s computed from different initial stock levels as long as the shocks, the discount rate, the depreciation rate, and the curvature of benefits and costs remain the same.

Optimal Quantity Policy. We begin by assuming a quantity solution $\{q_t^*\}$ to the original optimization problem

\[
\max_{\{q_t\}} \sum_{t=1}^{\infty} \mathbb{E}[L_t] \text{ where } L_t = B(S_t) - C(q_t) - \lambda_t (S_t - S_{t-1}(1-\delta) + \bar{q} + q_t),
\]  

(18)

where we assume, as before, quadratic forms for both benefits and costs:

\[
B(S_t) = b_0 + b_1 - \frac{1}{2}b_2 S_t^2
\]

\[
C(q_t) = c_0 + (c_1 + \theta_t)q_t + \frac{1}{2}c_2 q_t^2.
\]

Other than the concavity of benefits and the convexity of costs, we make no additional assumptions about parameters or the initial value $S_0$ other than the existence of a solution to Equation (18).\(^{19}\)

Corresponding Price Solution. Assuming a quantity solution exists, we can show that the optimal price policy is one which sets $P_t^* = c_1 + c_2 q_t^*$. Because $q_t(P) = \frac{P - c_1 - \theta_t}{c_2}$ under a

\(^{19}\) The quadratic assumptions concerning benefits is less innocuous here, however, where the stock level may vary considerably.
price policy, the first order conditions solving Equation (18) for a price policy will be almost the same as those under a quantity policy, except rather than solving

\[
E \left[ \frac{\partial}{\partial S_t} \sum_{i=1}^{\infty} \frac{L_i}{(1 + r)^i} \right] = 0
\]

\[
E \left[ \frac{\partial}{\partial q_t} \sum_{i=1}^{\infty} \frac{L_i}{(1 + r)^i} \right] = 0
\]

we instead solve

\[
E \left[ \frac{\partial}{\partial S_t} \sum_{i=1}^{\infty} \frac{L_i}{(1 + r)^i} \right] = 0
\]

\[
E \left[ \frac{\partial}{\partial q_t} \sum_{i=1}^{\infty} \frac{L_i}{(1 + r)^i} \right] = 0
\]

Because all the conditions are linear in the variables \( q_t \) and \( S_t \), and since \( q'_t(P_t) \) is a constant \( 1/c_2 \), the second set of conditions will hold as long as \( P_t^* \) is chosen such that \( E[q_t(P_t^*)] = q_t^* \) and \( E[S_t] = S_t^* \). That is, if \( P_t^* = c_1 + c_2 q_t^* \).

**Relative Advantage Measure.** We note that the relative advantage measure \( \Delta \) can be written as

\[
\Delta = \sum_{t=1}^{\infty} E \left[ \frac{B(S_{t, price}) - B(S_{t, quantity}) - C(q_{t, price}) + C(q_{t, quantity})}{(1 + r)^t} \right]. \tag{19}
\]

Because the expected value of both \( q_t \) and \( S_t \) under price controls equals the fixed values of \( q_t \) and \( S_t \) under quantity controls we can rewrite Equation (19) as

\[
\Delta = \sum_{t=1}^{\infty} E \left[ \frac{\frac{1}{2}b_t dS_t^2 - \theta_t dq_t - \frac{1}{2}c_2 dq_t^2}{(1 + r)^t} \right]. \tag{20}
\]
where $dq_t = q_{t, price} - q_t^*$ and $dS_t = S_{t, price} - S_t^*$ are the deviations under the price control from their expected value under the price control (which is the same as the optimal quantity control). Noting that $dq_t = -\frac{\theta_t}{c_2}$ and $dS_t = -(1-\delta)^{-1}\frac{\theta_t}{c_2} - (1-\delta)^{-2}\frac{\theta_t}{c_2} \cdots \frac{\theta_t}{c_2}$, we have that $\Delta$ depends only the cost shocks, the depreciate rate, the discount rate, and the curvature parameters $b_2$ and $c_2$, and not the initial stock level. Therefore the expression for $\Delta$ obtained from two different problems, one starting at the steady state stock level $\hat{S}$, and one starting at an arbitrary level $S_0$, but otherwise with the same aforementioned parameter values, will be identical. Since we have already obtained the solution from the steady state case in Equation (10), this will be the answer for all initial stock levels.\footnote{Alternatively, one can simply show that the expression in Equation (20) reduces to the expression in Equation (10).}

A.5 An Argument Favoring Price-Based Policies for Stock Externalities

At the steady state stock $\hat{q} = \delta \hat{S} - \bar{q}$ and expected marginal costs in one period will equal discounted and depreciated marginal benefits in all future periods:

$$E\left[ B'(\hat{S}) \frac{1+r}{r+\delta} - C'(\hat{q}) \right] = 0,$$

or after substitution and rearrangement,

$$(b_1 - b_2 \hat{S}) \frac{1+r}{r+\delta} = c_2 (\delta \hat{S} - \bar{q})$$

$$b_1 \frac{1+r}{r+\delta} + c_2 \bar{q} = (b_2 \frac{1+r}{r+\delta} + c_2 \delta) \hat{S}. \tag{21}$$

In Equation (21), the left-hand side can be viewed as the difference between the marginal costs and benefits (discounted and depreciated) of holding $S$ constant at zero. The coefficient on the right hand side indicates the rate at which this difference is reduced as $S$ is increased. Note that our focus on a bad stock implies $b_1$ is negative, so this is indeed the difference between the marginal cost of $\bar{q}$ gross reductions and the sum of discounted/depreciated marginal benefits when $S$ is zero.\footnote{$B'(0)$ is negative representing the negative marginal benefits associated with increasing the stock and positive marginal benefits associated with decreasing it.}
We now solve Equation (21) for the expression $\hat{S}/\hat{q}$:

$$\frac{b_1}{\hat{q}} \left( 1 + \frac{r}{r + \delta} \right) + c_2 = \left( \frac{b_2}{\hat{q}} \left( 1 + \frac{r}{r + \delta} + c_2 \delta \right) \right) \frac{\hat{S}}{\hat{q}}$$

$$\frac{\hat{S}}{\hat{q}} = \frac{b_1}{b_2} \left( 1 + \frac{r}{r + \delta} \right) + \frac{1}{c_2}$$

The first term on the right-hand side is negative since $b_1$ is negative for a bad stock. Assuming that the optimal steady-state stock is larger than the annual exogenous flow, this implies that $\delta + b_2 \frac{(1+r)}{(r+\delta)} / c_2$ is less than one and in particular that $b_2 \frac{(1+r)}{(r+\delta)} / c_2$ is less than one. In the absence of correlation among the cost shocks, this is *almost* the condition for prices to be preferred to quantities, namely $b_2 \left( \frac{1+r}{r + 2\delta - \delta^2} \right) > c_2$.

This difference arises because in computing conditions for an optimum, we are interested in matching up discounted and depreciated changes in the stock. In computing the differences between prices and quantities, we look at discounted and depreciated squared deviations in the stock. The squared deviations depreciate at a different rate, namely $2\delta - \delta^2$, compared to the deviations themselves at rate $\delta$. We can correct for this difference by noting that $\frac{2\delta - \delta^2}{\delta} \leq \frac{3}{2}$ for all $\delta$ between zero and one. If we assume that $\hat{S}/\hat{q} > 3/2$, we have

$$\delta + \frac{b_2 \frac{(1+r)}{(r+\delta)}}{c_2} < \frac{3}{2}$$

$$\frac{b_2 \frac{(1+r)}{(r+\delta)}}{c_2} < \frac{3}{2},$$

$$\frac{b_2 \frac{(1+r)}{(r+2\delta-\delta^2)}}{c_2} < 1$$

or that prices controls are preferred to quantity controls in the absence of cost correlation (based on Equation (10)). The only assumptions we have made are that: (i) costs and benefits are globally quadratic; (ii) marginal costs are zero when the produced flow is zero; (iii) the optimal stock level is larger, by a factor of 50 percent, than the annual exogenous flow; and (iv) the
marginal benefits associated with stock reductions are non-negative when the stock is zero (i.e., increases in the stock are bad even when the stock is zero).
References


