Optimal-Sustainable Management of Multi-Species Fisheries: Lessons from a Predator-Prey Model

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Abstract: In this paper we define fisheries management as sustainable if it maintains the net present value of the fishery. This definition is based on the principle of intergenerational fairness. If the sustainability of a fishery is held as a obligation by managers, then the traditional present-value maximization objective would be constrained by this obligation. Using a simple predator-prey model, we explore how the optimal-sustainable management of this fishery would differ from management that seeks to maximize the present value of the benefits. General lessons for fishery management are discussed.

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Theories of optimal management have played a useful role in applied fisheries management. While managers rarely attempt to apply such criteria exactly, these theoretical notions provide a useful guideline. Certainly the notion of *maximum sustainable yield* played this role for most of this century and *optimal sustainable yield*, in which both economic and biological benefits are recognized, is the guiding principle today (Roedel, 1975).

In most economic models, the criterion that lies behind prescriptions for optimal management is maximization of the present value of net benefits. Anderson (1986, 32) sums up the standard perspective, "Put succinctly, proper use of a fish stock requires that resources be utilized to exploit it such that the present value of future net returns is maximized." This perspective is reflected in virtually all economic models of fishery management (e.g., Clark 1976, Wilen 1985, Ströbele and Wacker 1995 and Kimel 1996).

In contrast to the economic models, the maximization of net benefits does not seem to be the only policy objective that motivates most fisheries management (Charles 1994). The terms that are used to describe appropriate fisheries management today de-emphasize the efficient management of a stream of economic benefits and focus on the issue of *sustainability* of the fishery. For example, consider the current interest in *ecosystem management*. Definitions of ecosystem management almost uniformly include sustainability among the

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array of policy objectives\textsuperscript{1} and Christensen et al. (1996, 666) state, "sustainability must be the primary objective" of ecosystem management.

When sustainability is introduced as a policy objective, however, the standard models used by economists are no longer directly applicable. For example, it is well established that under extreme circumstances the "optimal" fishery management can involve driving resource to extinction (Clark 1973). More generally, there is nothing in the present-value criterion that ensures that anything will be sustained.

In this paper we will propose a definition of sustainable fisheries management that is appropriate in the context of a multi-dimensional resource. We will argue that this framework provides useful conceptual guidance for fisheries managers seeking to achieve both efficiency \textit{and} sustainability. We apply the model to the theoretical problem of the management of a two-species fishery involving a predator-prey relationship. We then discuss how management strategies would differ depending on whether the goal is to maximize the present value of net benefits or to maximize those benefits subject to a sustainability constraint.

Results of this theoretical exercise will help us to address a number of questions that surround the issue of sustainable fisheries management: What is it that should be sustained? What sorts of tradeoffs between the species in the fishery are admissible? We close with a discussion of how the model sheds light on the problems faced by fisheries management today.

\textsuperscript{1} Schramm and Hubert (1996, p. 6) list four definitions of ecosystem management, two of which include the word, "sustainable," and a third that calls for protection the long-term integrity of the system.
An economic definition of sustainability

The most basic question that any definition of sustainability must answer is, What is that should be sustained? The fact that this question has no immediately obvious answer is apparent in the breadth of definitions of sustainability and sustainable fisheries management (e.g., WCED 1987, Charles 1994). At the heart of virtually all concerns about sustainability is a notion of intergenerational fairness or equity (see Pezzey, 1989). Hence, the definition of sustainable fisheries management that we use is based on this principle.

The notion of fairness has been studied extensively by economists (Foley 1967, Varian 1974, Pazner and Schmeidler 1978, Baumol 1986). The pervasive principle in this literature is that an allocation is fair "if and only if each person in the society prefers his [or her] consumption bundle to the consumption bundle of every other person in the society" (Foley 1967, 74). There are two important components to this definition. First, is that fairness implies a lack of envy -- if an allocation is completely fair then no agent would be envious of the allocation of any other agent. Second, there is an implicit assumption that some elements of the endowment might be substitutes for others so if two agents have very different allocations it still may not be true that neither agent envies the other.

We now consider how the principle of fairness might be applied in the intertemporal context of a fishery. Fisheries are valued because they make possible a stream of benefits to society including fish, recreation and a range of ecosystem services. We value fisheries not only because of the benefits that they can generate immediately but because of the promise of what they will provide us in the future. The value of a fishery at a particular point in time, therefore, is a measure of the stream of benefits that can be generated given the state of the
fishery at that time. If this value declines over time, then stakeholders that come later in time
will be envious of previous stakeholders. Hence, if current management leads to a fall in the
value of the fishery, then it follows that those management practices are unfair to future
stakeholders in the fishery or, as we will use the term, inconsistent with sustainability.

The notion of value used above is general but is not sufficiently precise to be directly
operational. In this paper we employ the standard economic interpretation of value based on
the discounted present value of the benefits (Anderson 1986). That is, the value of a fishery,
$V(x)$ is defined as the present value of the expected stream of surplus generated by the fishery,

$$V(x_0) = \max_{\{x_t, z_t, b\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(x_t, z_t)$$

where $x_t$ is the state of the fishery in period $t$, $z_t$ is the vector of management decisions made in
period $t$, $\beta$ is the discount factor$^3$, $E_t$ is the expectation operator based on the knowledge in $t$,
and $U(\cdot)$ is a function that captures economic benefits achieved in the fishery during the period
$t$. In the examples below we consider only the surplus generated by commercial harvests from
the fishery. Other market and non-market benefits could, in principle, be incorporated into the
valuation of the intratemporal benefits.

Given the interpretation of sustainability that we have provided above and our choice
of a measure of value, an operational definition of sustainable fisheries management follows.
Namely, the choices in period $t$ are said to be consistent with sustainability if

$$V(x_t) \leq E_t V(x_{t+1})$$

(1)

$^2$ See Thomson and Varian (1985) for a review of this literature.

$^3$ If $\rho$ is the discount rate, the $\beta=1/(1+\rho)$. 
If a fishery manager adheres to a binding obligation of sustainability, then, in addition to the present value criterion, managers would impose upon themselves a constraint to ensure that (1) is achieved. The optimization problem solved by the manager in period $t$ therefore, can be written in the recursive form,

$$V^s(x_t) = \max_{z_t} E_t U(x_t, z_t) + \beta V^s(x_{t+1}) \text{ s.t.}$$

$$E_t U(x_t, z_t) + \beta V^s(x_{t+1}) \leq E_t V^s(x_{t+1})$$

where the value function is written $V^s(\cdot)$ to indicate differentiate it from the value function that arises from the unconstrained maximization of the present value of the benefits.

The solution of dynamic optimization problems are difficult in general and the sustainability-constrained optimization problem, (2), is no exception. Because a closed form analytical solution for most dynamic optimization problems is not possible, numerical methods are required to approximate the solution (Rust 1996, Judd 1996). Woodward (1997) shows that a numerical algorithm of successive approximation of $V^s(\cdot)$ can be used to sustainability-constrained optimization problems. A discussion of the details of the algorithm and the restrictions that must be imposed on problems in order to allow their solution is provided in an appendix.

In the remainder of this paper we apply the notions of sustainability-constrained optimization that we have proposed above to the case of a simple predator-prey fishery. While the model is intended to be heuristic and is not empirically based on any particular fishery, the patterns that we find would be most directly translatable to fisheries in which two commercial species dominate.
Description of the model under certainty

The model that will serve as the basis for our discussion is composed of two species that interact in a predator-prey relationship. Both the prey species, $x_1$ and the predator, $x_2$, are harvested commercially.\footnote{While no specific units are used here, $x_1$ and $x_2$ are measures of the biomass of the two species.} Effort expended on the $j^{th}$ species in period $t$, $z_{jt}$, produces harvests,

$$ h_{jt} = x_{jt} \left( 1 - e^{-q_j z_{jt}} \right) $$

where the parameter $q_j$ indicates the effectiveness of effort in capturing $j^{th}$ species. We assume that indirect harvests or bycatch are negligible. The stocks that remain after harvesting evolve according to the predator-prey model of Clark (1985, p. 195):

$$ x_{1t+1} = (x_{1t} - h_{1t}) \left[ 1 - m_1 - \alpha (x_{2t} - h_{2t}) \right] + r $$

and

$$ x_{2t+1} = (x_{2t} - h_{2t}) \left[ 1 - m_2 + \phi \alpha (x_{1t} - h_{1t}) \right] . $$

The parameters $m_1$ and $m_2$ are the autonomous mortality rates of each species while $r$ is the rate of density-independent recruitment for the prey species. The predator-prey interactions are determined by the parameters $\alpha$ and $\phi$ with $\alpha (x_{2t} - h_{2t})$ is the percent of the predator population that is consumed in any given period and $\phi$ is the rate at which the predator converts the prey into its own biomass.

For the unexploited fishery there are two equilibria:

$$ x_1 = r/m_1, \; x_2 = 0 \text{ and } x_1 = m_2/\phi \alpha, \; x_2 = r \phi / m_2 - m_1 / \alpha. $$
However, when fishing effort is introduced, \( h_i > 0 \), the steady-state values are substantially more complicated. When \( x_2 = 0 \), \( x_1 \) remains constant if

\[
    x_1 = \frac{r \cdot e^{q_{z_1}}}{m_1 + e^{q_{z_1}} - 1}.
\]

An interior steady state occurs at

\[
    x_1 = x_1^{SS}(z) = \frac{e^{q_{z_1}}(-1 + e^{q_{z_2}} + m_2)}{\alpha \lambda}, \quad \text{and}
\]

\[
    x_2 = x_2^{SS}(z) = \frac{e^{q_{z_2}}}{\alpha(-1 + e^{q_{z_2}} + m_2)} \left(-1 + e^{q_{z_1}} + e^{q_{z_2}} - e^{q_{z_1} + q_{z_2}} + m_1 - m_1 e^{q_{z_2}} + m_2 - m_2 e^{q_{z_1}} - m_1 m_2 + \alpha \lambda r\right).
\]

For reasons of simplicity, we consider the simple case in which the fishery is valued only for the profits that it is able to generate in each period, i.e.,

\[
    U(\cdot) = \pi(x_1, z_1, x_2, z_2) = p_1 h_1 + p_2 h_2 - c(x_1, z_1, x_2, z_2),
\]

where \( p_1 \) and \( p_2 \) are fixed prices of the two species per unit of output and \( c(\cdot) \) is the total cost of obtaining the harvests \( h_1 \) and \( h_2 \). Cost is assumed to be a linear in effort and increasing and concave function in the stock,

\[
    c(x_1, z_1, x_2, z_2) = c_1 x_1^T z_1 + c_2 x_2^T z_2.
\]

This specification allows costs to increase as harvests rise, even if effort is held constant.\(^6\)

Using the standard present-value criterion, the manager's objective is

\[\text{\textsuperscript{5}}\text{The values of the parameters used in the numerical simulations below and presented in Table 1 were chosen specifically to yield steady-state unexploited stocks of 4.0 and 2.0 for } x_1 \text{ and } x_2 \text{ respectively.}\]

\[\text{\textsuperscript{6}}\text{This form has the advantage of allowing costs to rise as harvests increase due to increases in the stock. However, it has the disadvantage of that for sufficiently small stock levels, increases in the stock can actually lead to a reduction in profits.}\]
\[
\max_{\{x_t, z_{t}\}, t=0} \sum_{t=0}^{\infty} \beta^t \pi(x_t, z_{1t}, x_{2t}, z_{2t}) \text{ s.t. (3) and (4)}.
\]

The value of the stock at time \( t=0 \) can therefore be written recursively in the form of Bellman's equation,

\[
V(x_{10}, x_{20}) = \max_{z_{10} \in z_{20}} \pi(x_{10}, z_{10}, x_{20}, z_{20}) + \beta V(x_{1t}, x_{2t}) \text{ s.t. (3) and (4)}. \tag{8}
\]

Policies that satisfy this objective will be called PV-optimal.

While our eventual interest is to identify the full set of policies that would arise across the state space, solving for the PV-optimal steady state can be a useful first step in identifying the nature of those policies. Making standard regularity assumptions, the optimal choices, \( z_{10} \) and \( z_{20} \), can be obtained by first order conditions of (8)

\[
\frac{\partial V}{\partial z_i} = \frac{\partial U}{\partial z_i} + \sum_j \left( \frac{\partial V}{\partial x_{j+1}} \frac{\partial x_{j+1}}{\partial z_i} \right) = 0, i = 1, 2. \tag{9}
\]

Let \( \lambda_{1t} = \frac{\partial V}{\partial x_{1t}} \), so that

\[
\lambda_{1t} = \frac{\partial U}{\partial x_{1t}} + \sum_j \left( \lambda_{j+1} \frac{\partial x_{j+1}}{\partial x_{1t}} \right). \tag{10}
\]

At the steady state \( \lambda_{jt} = \lambda_{j+1} = \lambda_{j} \) so that (9) and (10) can be rewritten,

\[
\frac{\partial V}{\partial z_i} = \frac{\partial U}{\partial z_i} + \sum_j \left( \lambda_j \frac{\partial x_{j+1}}{\partial z_i} \right) = 0, i = 1, 2 \tag{11}
\]

and

\[
\lambda_i = \frac{\partial U}{\partial x_{it}} + \sum_j \left( \lambda_j \frac{\partial x_{j+1}}{\partial x_{it}} \right) i = 1, 2. \tag{12}
\]
Solving (6), (7), (11) and (12) simultaneously for \(x_1, x_2, z_1, z_2, \lambda_1, \lambda_2\) it is possible to identify the steady state equilibrium. When no interior solution is optimal, then the equations \(x_2=0\) and (5) would be substituted for (6) and (7).\(^7\)

In Table 1, we present the steady-state PV-optimal values of \(x_1, x_2, z_1, z_2, h_1\) and \(h_2\) given the parameter values listed in the table. We also present the elasticities of each of these variables with respect to changes in the parameters. The most striking result in this table is that the elasticities are quite high -- changes in the parameters' values would lead to significantly different results at the steady state. This is particular true of harvest rates and, at the extreme, a one-percent increase in the price of the prey would lead to a 12% increase in steady-state harvests of that species and 9% decrease in harvests of the predator. The high elasticities with respect to the biological parameters are particularly disturbing since the uncertainty surrounding these parameters is likely to be significant. While the magnitude of the relationships between the variables and the parameters is quite sensitive to the actual parameter values, the sign of the elasticities was consistent over a wide range of values.

\(^7\) For some parameter values the solution to this system of equations yields values with \(z_1\) or \(z_2\) less than zero. To avoid this the Kuhn-Tucker conditions would need to be included in the problem. This complication, however, adds little to our understanding of the questions under consideration in this paper.
Table 1
Numerically calculated elasticities of steady state stocks and effort levels with respect to the model parameters


table 1

<table>
<thead>
<tr>
<th>steady state values</th>
<th>$x_{11}^{SS}$</th>
<th>$x_{21}^{SS}$</th>
<th>$z_{11}$</th>
<th>$z_{21}$</th>
<th>$h_{11}$</th>
<th>$h_{21}$</th>
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<td></td>
<td>1.27</td>
<td>0.60</td>
<td>0.09</td>
<td>0.05</td>
<td>0.11</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Elasticities

$(\% \Delta \text{ in variable} / \% \Delta \text{ in parameter})$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base case values</th>
<th>$\varepsilon_{x_{11}}$</th>
<th>$\varepsilon_{x_{21}}$</th>
<th>$\varepsilon_{z_{11}}$</th>
<th>$\varepsilon_{z_{21}}$</th>
<th>$\varepsilon_{h_{11}}$</th>
<th>$\varepsilon_{h_{21}}$</th>
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<td></td>
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<td></td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.48</td>
<td>-0.56</td>
<td>-0.32</td>
<td>-1.89</td>
<td>-1.76</td>
<td>-2.37</td>
<td>-2.06</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.11</td>
<td>0.08</td>
<td>-1.77</td>
<td>3.89</td>
<td>-4.64</td>
<td>3.80</td>
<td>-6.34</td>
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<tr>
<td>$r$</td>
<td>0.80</td>
<td>0.76</td>
<td>0.28</td>
<td>3.99</td>
<td>1.86</td>
<td>4.58</td>
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<tr>
<td>$\alpha$</td>
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<td>-0.21</td>
<td>2.08</td>
<td>-5.51</td>
<td>5.65</td>
<td>-5.49</td>
<td>7.64</td>
</tr>
<tr>
<td>$\phi$</td>
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<td>-0.09</td>
<td>2.34</td>
<td>-5.47</td>
<td>6.19</td>
<td>-5.32</td>
<td>8.45</td>
</tr>
<tr>
<td>$q_1$</td>
<td>1.00</td>
<td>-0.21</td>
<td>-2.59</td>
<td>11.25</td>
<td>-5.68</td>
<td>10.56</td>
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<tr>
<td>$q_2$</td>
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<td>-1.24</td>
<td>1.16</td>
<td>-0.77</td>
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<tr>
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<td>-0.04</td>
<td>0.35</td>
<td>-0.27</td>
</tr>
</tbody>
</table>

The steady state values, however, do not tell the whole story. Once the fishery is at the steady state the PV-optimal policy leaves the fishery unchanged and, therefore, satisfies the sustainability criterion from that point onward. Since our principal concern in this paper is the consideration of the sustainability of policies, we will be more interested in policies at points away from the steady state.

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8 The elasticities were obtained based on the solutions to the system of steady state equations at the base-case parameter values and at values both 1% below and 1% above. The average elasticity over both the positive and negative shocks is presented here.

9 The base-case parameter values were chosen only because their relative magnitudes seemed plausible and because the led to interior solutions for the variables $x_1, x_2, z_1$ and $z_2$. They are not intended to be parameters of any true fishery.
To evaluate policies away from the steady state, we solved for the full array of policies using a successive approximation algorithm. The details of the numerical methods used to solve for the PV-optimal and S-optimal policies are elaborated in an appendix. Figure 1 presents a variety of paths within the state space that follow from PV-optimal policies. From all points in the state space the PV-optimal management leads toward the steady-state levels indicated in Table 1. Adjustment to the steady state is quite rapid with most paths reaching within one-percent of the steady state in less than ten periods although adjustment is significantly slower if the initial stocks are quite low. When the fishery is heavily stocked with both species (the northeast corner of the figure) the PV-optimal strategy leads to rapid reductions in both the predator and the prey. At the other extreme, in the southwest corner of the figure the optimal policy leads to accumulation in both stocks, with the move in $x_1$ taking place first followed by the growth in $x_2$. 
When the fishery begins in either the northwest or southeast corners, where relative to the steady state one of the species is overstocked and the other is understocked, then the optimal policy leads to reductions in the one species and increases in the other. It is at these points where the question of sustainability is most interesting—when is this substitution of one element of the asset vector for the other consistent with sustainability? We can begin to answer this question by looking at the PV-optimal value function, \( V(x_1, x_2) \), in Figure 2. This surface shows the present value of the stream of net benefits that can be obtained starting at each point in the state space. The value function is monotonically increasing in \( x_1 \) so that, holding \( x_2 \) constant, policies that lead to positive changes in \( x_1 \) are consistent with sustainability, while those that lead to reductions in \( x_1 \) will be inconsistent with sustainability. Due to the predator's dual asset-nuisance characteristic, however, \( V(x_1, x_2) \) is not monotonic in \( x_2 \). When predator stocks are low, the total value of the fishery actually declines as the...
biomass of the predators increases because the deleterious effect on the prey species outweighs the benefits to the predator fishery. Only after the stock reaches a level above about 0.5 do increments to that stock actually make the aggregate fishery more valuable.

Figure 2

Value function, \( V(x_1, x_2) \) for unconstrained optimization problem

Figure 2 is helpful in beginning to determine which of the paths in Figure 1 are consistent with sustainability. As long as \( x_2 \) is above 0.5, any path that leads to increments in both species will be consistent with sustainability. As seen by the contours of the value function in the figure, if \( x_2 \) is above 0.5 then slight reductions in the predator stock can be sustainable if there is a coincident increase in the stock of the prey. Likewise, we see that significant reductions in \( x_1 \) can be consistent with sustainability if \( x_2 \) grows slightly. When the predator stock is below 0.5, then the predator's nuisance quality predominates and growth in the stock of this species actually leads to a decline in the value of the fishery. Hence, some of the paths in Figure 1 that lead to increments to both stocks actually reduce the value of the fishery as it approaches the steady state.
Efficient sustainable management of the fishery under certainty

We now turn to the discussion of policies when the fishery agency chooses to obligate itself to satisfy the sustainability criterion. In this case the management agency would seek to maximize the present value of the harvests subject to the sustainability constraint,

\[ E_t V(x_{t+1}) \geq V(x_t). \] (1)

There is no a unique steady state to the sustainability-constrained problem and it is not possible to find an analytical solution to this problem. Rather, these problems must be solved numerically. The details of the successive approximation algorithm used to obtain numerical solutions are discussed in an appendix.

![Figure 3](image)

**Figure 3**

Time paths of the predator and prey species in the sustainability-constrained deterministic model (base-case parameters -- see Table 1)

In Figure 3 we present a variety of time paths that follow from the S-optimal policy rule. Comparing this figure with Figure 1 we see how the introduction of the sustainability constraint alters the management of the fishery. The S-optimal paths that begin with \( x_2 \) below the PV-optimal steady state end at essentially the same point, though the route taken differs
somewhat because of the non-monotonic portion of $V$ noted above. When $x_2$ starts above 0.5, however, the S-optimal paths follow along iso-value function contours until a steady state is reached. The actual steady state that is reached is a function of the initial stocks and the locus of steady states slopes from the PV-optimal steady state in a northwesterly direction. Figure 3 clearly shows the meaning of sustainability in a multidimensional resource for it identifies where tradeoffs are possible and can be exploited.

In figures 4 and 5 we present the PV- and S-optimal harvest levels as functions of the stocks, $x_1$ and $x_2$. We see that the introduction of the sustainability constraint has significant effect on the policy choices for both variables. Harvests of the prey in the sustainability-constrained model actually exceed that of the PV-optimal policies when the predator stock is high and the prey stock is low. However, when the predator stock is abundant, then the S-optimal policy is to refrain from harvesting the prey in order to protect the food source for the relatively valuable predator.
The harvests of the predator, presented Figure 5, demonstrate that the introduction of the sustainability constraint has a dramatic affect on the harvest levels of this species. In the unconstrained model the optimal policy was to quickly and drastically reduce the predator stocks. For example, at high levels of $x_2$ the PV-optimal policy is to harvest up to 60% of the stock in a single period. When the sustainability constraint is introduced, however, the optimal policy is to harvest this species much more cautiously, maintaining the stock levels and, therefore, the valuable harvests over the long run. Recall from Table 1 that at the PV-optimal steady state harvests of the predator were 0.02. For some initial endowments, the sustainability-constrained model leads to long term harvests of nearly twice that level. Hence, the preservation of the predator stocks that can be seen in Figure 3 has an important consequence in significantly increasing harvests of that species.
Particularly given the harvests levels presented in Figure 5, it is not surprising to find that the introduction of the sustainability constraint comes at a significant cost. This cost is visible in two sections of Figure 6, where $x_2$ is both low and high. While the PV-optimal value of the fishery (Figure 6A) increases sharply as the predator stock rises, this increment to the value is captured by current users and future users of the fishery are left with a substantially less valuable resource. If managers treat both current and future generations equally (Figure 6B), then they must forego the significant rents that can be obtained because of the abundance of the predator. Hence, once the sustainability constraint binds, increments to the predator stock lead to increasingly small gains in the value of the fishery.

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10 In a separate paper that can be obtained from the authors we show that the introduction of a sustainability constraint can be inefficient in the sense that both current and future generations can be made better off by relaxing the constraint. In this model there is evidence that the norm of sustainability is inconsistent with Pareto efficiency when the stock of either the predator or the prey is large.
Discussion of the numerical results

While the exact patterns exhibited in the figures above are specific to the model under consideration, there are some important patterns. First, as is seen in the policies presented in figures 4 and 5, the sustainability constraint significantly complicates the problem. While interactions need to be taken into account, the PV-optimal policies are largely a function of one species or the other; single-species models may not be too far off the mark in determining the appropriate policy in this model. However, when the sustainability constraint binds, however, the S-optimal policy for each species is intimately related to stock of both species. Figures 4 and 5 demonstrate that, particularly for the prey harvests, the S-optimal policy must take into account not only the direct effect on the species being managed, but their dynamic interaction and how the two policies jointly affect the combined values of the fisheries in the future. The lesson here seems quite general.

The second principal conclusion that one can draw from the analysis is that sustainability is not simply about the preservation of each resource. For virtually all the paths presented in Figure 3, the optimal sustainable policies led to a decline in one or the other of the stocks. We see, therefore, that optimal sustainability will almost always involve tradeoffs. Furthermore, as is seen in the comparisons of the constrained and unconstrained policies, a sustainability constraint can lead to significant and surprising results. Identifying the optimal-sustainable policy will likely involve more than simple rules of thumb.

Finally, we find that the S-optimal paths do not lead to a unique steady state as do the PV-optimal paths in Figure 1. Where a sustainability-constrained path ends depends critically upon where it started. This draws an important distinction between management in a sustainability-constrained economy and the unconstrained PV-maximizing management of a
fishery. In PV-optimal management the principal concern is to find the unique equilibrium and then "any 'reasonable' method of reaching [that point] would probably be close to the optimum" (Clark 1985, 198). Since sustainability-constrained fishery management does not lead to a unique steady state, the policy objectives in this case are fundamentally altered.

Conclusions

We began by suggesting that the fundamental value of theoretical fisheries models is that they provide some general guidance to policy makers. We should stop, therefore, to consider what guidance is provided by the sustainability-constrained model introduced here and how this differs from that of the standard present-value maximization model.

First, we provide a new interpretation of the term *sustainability* that is established based on the normative foundation of intergenerational fairness. Whether one agrees that this is the appropriate goal for fisheries management or not, we hope that it will at least clarify what is sought when sustainability is put forth as a policy objective. Hence, this interpretation of the meaning of sustainability provides one answer to the questions: What should be sustained and what tradeoffs are admissible? As such, we provide a benchmark against which alternative criteria of sustainable fisheries management can be evaluated.

How might fisheries policies in a sustainability-constrained management model differ from management intended to maximize the present value of the resource? First, the sustainability constraint adds an important additional interaction between the two species. This fundamentally alters policy. S-optimal policies must take into account not only the biological interactions between the two species, but also how the two species work jointly to sustain the value of the fishery. Sustainability requires, therefore, that policy makers avoid the
p piecemeal approach to resource management problems. We found that PV-optimal harvests of the predator led to rapid depletion of these stocks. Such seemingly myopic behavior has long been subject of criticism from those concerned with sustainability. However, we also found that the S-optimal harvests of the prey actually exceeded the PV-optimal harvests over a narrow range. If economic sustainability is proposed as a policy goal, therefore, its implications will not always be immediately clear.

The pursuit of sustainability has long been an objective of fisheries management. However, when the fishery in question is made up of multiple economically important species the meaning of sustainability is not clear. What should be sustained and why? The framework in this paper provides one possible set of answers to this question based on the principle of intergenerational fairness. While the fishery model that we use is extremely simplistic, it would be conceptually straightforward to include additional elements in the endowment vector, non-market benefits from the species and/or uncertainty. The general framework provides, therefore, a quite flexible structure in which to study the issue of sustainable management.
Appendix

Numerical methods used to solve for the PV-optimal and S-optimal policies in the predator-prey model

The results presented in this paper were generated by a numerical algorithm that approximately finds the PV- and S-optimal policies. In this appendix we briefly describe the algorithm used to solve both problems and the numerical methods used.

The PV-optimal policies are found by the process of successive approximation of the value function (Bertsekas 1976). That is, using an arbitrary initial estimate of the value function \( V^1(x_t) \) in the first stage of the algorithm, the problem

\[
V^2(x_t) = \max_{z_t} U(x_t, z_t) + \beta E_t V^1(x_{t+1})
\]

is solved for a finite set of points in the state space, say \( X \). The bounds and the number of points in \( X \) are set in order to ensure the greatest accuracy possible within a reasonable amount of computational time. The set of values \( V^2(x_t) \) for all \( x_t \in X \) is then used in the next iteration to obtain the next estimate of the value function.

Since, the set of points in the state space is necessarily finite, some approximation method must be used to estimate the value of points not included in the set. That is, for most choices \( x_{t+1} \notin X \) so that the values obtained in the first stage can not be used to directly calculate \( V^2(x_{t+1}) \). A number of alternatives exist for estimating \( V(x) \) over the compete range of \( x \) including rounding, interpolation and parametric approximation. In this case we used a special kind of polynomial approximation using Chebyshev polynomials. Press et al. (1989) provide a careful discussion of the use of these polynomials. They also state why Chebyshev polynomials are particularly attractive:
The Chebyshev approximation is very nearly the same polynomial as the holy grail of approximating polynomials the *minimax polynomial*, which (among all polynomials of the same degree) has the smallest maximum deviation from the true function $f(x)$. (p. 149)

In all the cases presented above, a two-dimensional Chebyshev polynomial with 17 nodes in each dimension is used.

By following this successive approximation algorithm, it is possible to obtain increasingly more accurate estimates of the true value function, say $V^*(x)$. For discrete dynamic programming, the successive approximation method is a contraction mapping (Bertsekas 1976) and the Chebyshev approximation method used here behaved similarly. If an algorithm is a contraction mapping then the difference between the value functions at each stage, $\varepsilon_k \equiv \max_x |V^k(x) - V^{k-1}(x)|$, declines from stage to stage. Furthermore, we know that $\varepsilon_k \leq \max_x |V^k(x) - V^*(x)|$. Hence, once $\varepsilon_k$ has reached an acceptable level, $V^k(x)$ is treated as if it were the true value function $V^*(x)$. The set of S-optimal policies is then obtained from the solution to the $k^{th}$ iteration of the algorithm.

The S-optimal policies and value function are obtained in a similar manner. Using $V^*(x)$ as an initial approximation of the sustainability-constrained value function, $V^S(x)$ is approximated by successively solving the sustainability-constrained optimization problem,

$$V^{n+1}(x_t) = \max_{z_t} U(x_t, z_t) + \beta E_t V^n(x_{t+1}) \text{ s.t.}$$

$$U(x_t, z_t) + \beta E_t V^n(x_{t+1}) \leq V^n(x_{t+1})$$

with $V^0(x) = V^*(x)$. Woodward (1997) shows that this successive approximation algorithm will converge to the true sustainability-constrained value function, $V^S(x)$. As in the unconstrained portion of the algorithm, the value function at each stage is approximated using Chebyshev polynomials. Since the sustainability-constrained portion of the algorithm is not a contraction
mapping, it is necessary to use a stricter conversion criterion in this portion of the algorithm to increase the likelihood that the algorithm is sufficiently close to the true value function. As with the PV-optimal case, the S-optimal policies are then taken from the solution of the last iteration of the successive approximation algorithm.

Once the PV- and S-optimal problems are solved, it is possible to trace out a policy paths that would follow from using the associated policy rules. The paths in figures 1 and 3 are obtained by first selecting an arbitrary set of starting points around the perimeter of the chosen state space then tracing out the PV-optimal and S-optimal paths from those points by solving the appropriate optimization problem at each point in the path. The bounds on the state space were chosen so that the all qualitatively interesting patterns are observed and so at a point where any expansion in the bounds does not alter the nature of the results. Strictly speaking, if optimal paths lead out of the defined state space, then there is no guarantee that the algorithm has reached the correct solution. This is problematic in the sustainability-constrained case because S-optimal paths for the problem at hand consistently left the square grid required by the Chebyshev polynomials, regardless of where the bounds were set. To increase our confidence in the results, a variety of bounds were tried in order to ensure that the patterns found were not a reflection only of bounds on X that have no economic content.
References


